Chapter 1

Mathematical review

1.1 Introduction

- Physics is based on experimental observations and quantitative measurements.
  - Observation: the story of Newton's apple.
  - Measurement: we can measure the falling time of the apple.
- The main objective of physics is to find the limited number of fundamental laws that govern natural phenomena and to use them to develop theories that can predict the results of future experiments.
- Physical laws are mathematical equations that relate physical quantities to each other.

Quantities we call quantity anything that can be measured like mass, density, temperature, electric field, etc.

Note: In physics, there are two type of quantities:

- **Scalar quantities** refers to number like length.
- **Vector quantities** refers to the magnitude and direction like force.

- In physics, there are seven quantities.
  1. Length: the distance between two points.
  2. Mass:
     - a num-ber we attach to each particle or body and that it is obtained by comparing the body with a standard body, using the principle of an equal arm balance. Mass, therefore, is a coefficient that distinguishes one particle from another
     - measures the inertia of body, meaning the resistance to acceleration (change of velocity) when a net force is applied.
  3. Time: the continued sequence of events that occurs in an apparently irreversible succession from the past, through the present, into the future.
  4. Electric current: Movement of charged body.
  5. Thermodynamic temperature: represent the measure of the average total internal energy of an object (measure of its kinetic energy, energy of motion).

6. Amount of substance: a quantity proportional to the number of elementary entities of a substance.
7. Luminous intensity: the quantity of visible light that is emitted in unit time per unit solid angle.

- All other quantities in physics can be expressed in terms of these basic quantities.

Dimension: The dimension of a quantity is simply its physical nature. A quantity can have the dimensions of a mass, speed, energy, etc.

Unities: The same length can be quantified differently from one country to another. For example, a distance between two points can be given in fingers, feet, yards, miles, and meters..., it is, therefore, necessary to define a universal unit so that the quantities have meaning for everyone.

In 1960, an international committee established standard units for the basic quantities. The system is called the "SI" system of units. SI is the abbreviation for “International System”).

Dimension	Symbol	Unity
Length	L	Meter (m)
Mass	M	Kilogram (Kg)
Time	T	Second (s)
Electric current	A	Ampere (A)
Thermodynamic temperature	ϴ	Kelvin (k)
Amount of substance	n	Mole (mol)
Luminous intensity	J	Candela (cad)

If X is a given quantity, the dimension of X will be denoted by [X]. for example, if X represent the volume, so $[V] = [L] x [L] x [L] = L^3$.

Dimensional analysis

As we said before, any quantity can be expressed in terms of the seven basic quantities. Then if we suppose a quantity X we can write:

$$[X] = [L]^{\alpha}. [M]^{\beta}. [T]^{\gamma}. [A]^{\varepsilon}. [\Theta]^{\mu}. [N]^{\chi}.[J]^{\omega} = L^{\alpha}. M^{\beta}. T^{\gamma}. A^{\varepsilon}. \Theta^{\mu}. N^{\chi}.J^{\omega}$$

The two member the above equation must be the same, so we can find the real values of α, β, γ, ε, μ, , and ω.

Examples :

- The time equation for uniform rectilinear motion is x = v.t where x is distance, v is speed and t is time. So,

$$\textbf{[Velocity] = [x / t] = [x] / [t] = L / T = L.T}^{-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

- The kinetic energy E of an object of mass m moving at speed v is written as Ec = ½ mv^2.

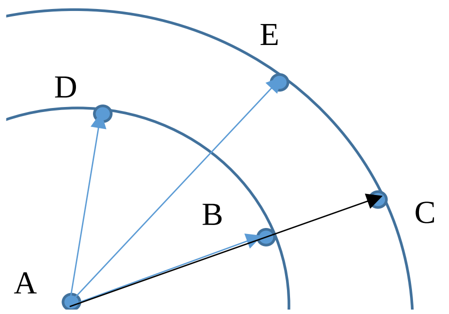
We then have,

$$\textbf{[Energy]= [E] = [1/2]. [m]. [v}^2\textbf{] = 1. M. (L.T}^{-1}\textbf{)}^2 \textbf{ = M.L}^2\textbf{.T}^{-2} \ldots\ldots\ldots\ldots\ldots\ldots 2$$

Note: the dimension of any number is 1.

1.2 Vector

Suppose that an object moves in a straight line from the initial point A to the final point B. This movement is characterized by the length of the segment AB and the direction from A to B. This quantity is represented by a vector $\overrightarrow{AB}$.



$\overrightarrow{AB} \neq \overrightarrow{AC}$: they have the same direction but not the module

$\overrightarrow{AB} \neq \overrightarrow{AD}$: they have the module but not the same direction

$\overrightarrow{AB} \neq \overrightarrow{AE}$: they have neither the same module nor the same direction

- **Unit vector:**

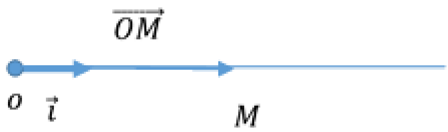
Unit vector only indicates the direction of the vector; its length is equal to one unity. Generally, the unit vector is represented by $\vec{u}$. Then, we can write $\overrightarrow{AB} = \|AB\|\vec{u}$.

$$\vec{u} = \frac{\overrightarrow{AB}}{\|AB\|},$$

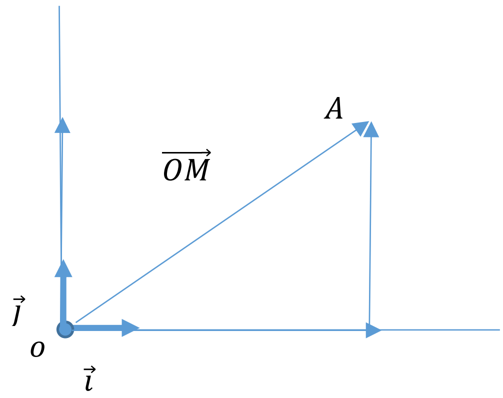
$$\|\vec{u}\| = 1$$

Graphical Representation of a vector :

- Cartesian Coordinate System $(\vec{\imath}, \vec{\jmath}, \vec{k})$



$$\overrightarrow{OM} = \|OM\| \times \vec{\imath}$$

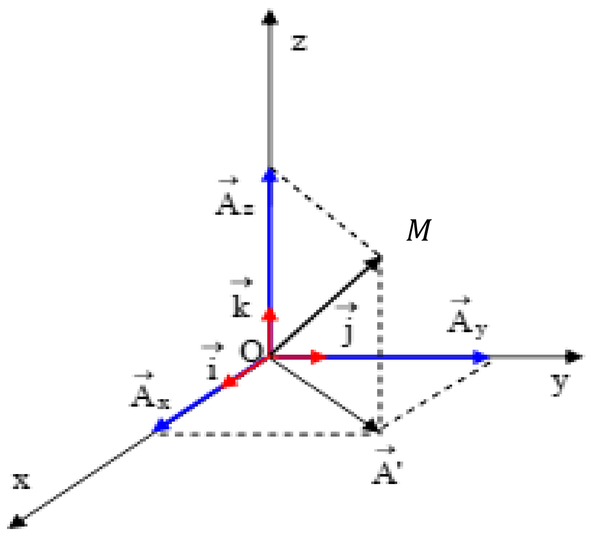


Vector in plane can be written in terms of two independent linear vectors

$$\overrightarrow{OM} = \overrightarrow{A_x} + \overrightarrow{A_y}$$

$$\overrightarrow{OM} = \|A_x\| \times \vec{\imath} + \|A_y\| \times \vec{\jmath}$$

$$\overrightarrow{OM} = \quad x\vec{\imath} + y\vec{\jmath}$$



Vector in space can be written in three non-parallel vectors, two by two, and not all of them in the same plane.

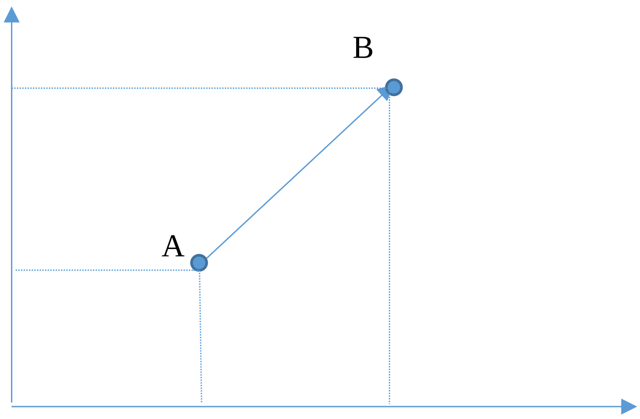
$$\overrightarrow{OM} = \overrightarrow{A_x} + \overrightarrow{A_y} + \overrightarrow{A_z}$$

$$\overrightarrow{OM} = \|A_x\| \times \vec{\imath} + \|A_y\| \times \vec{\jmath} + \|A_z\| \times \vec{k}$$

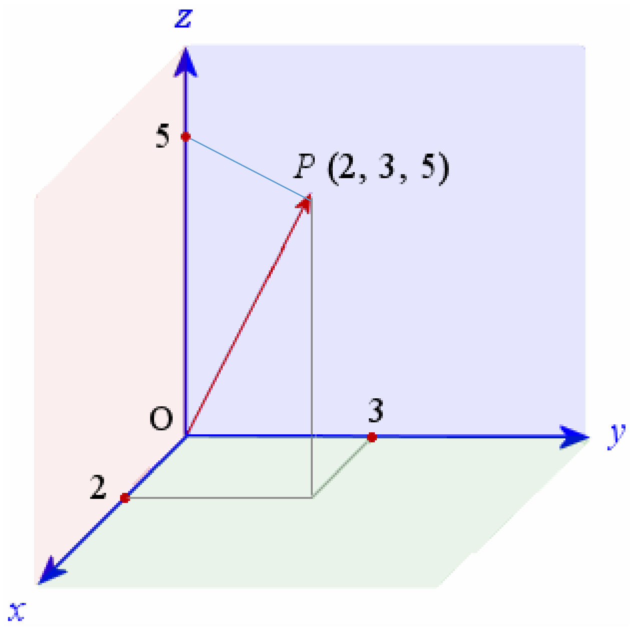
$$\overrightarrow{OM} = \quad x\vec{\imath} + y\vec{\jmath} + z\vec{k}$$

Vector can be defined in plane by the starting points A($x_{A,}y_{A,}$) and the ending B($x_{B,}y_B$), then ;

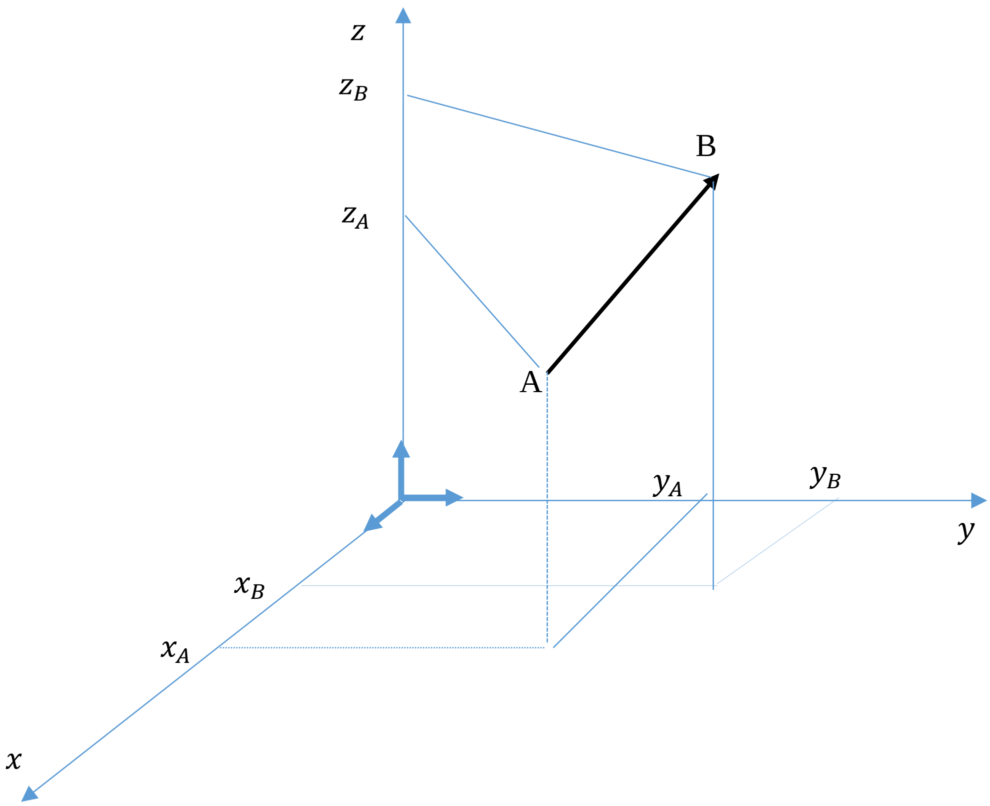
$$\overrightarrow{AB} = (x_B - x_{A)}\vec{\imath} + (y_B - y_{A)}\vec{\jmath} + (z_B - z_{A)}\vec{k}$$



Representation of a point in space



In space a vector can be defined by the starting points A(x_A, y_A, z_A) and the ending B(x_B, y_B, z_B), then $\overrightarrow{AB} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}$



Operations on vectors

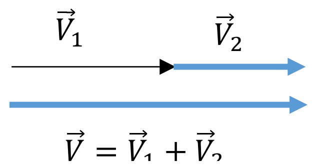
1. Addition:

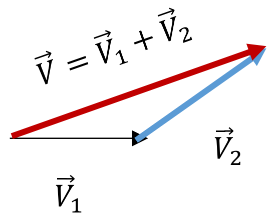
Consider

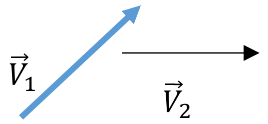
$\vec{V}_1 = \quad x_1\vec{i} + y_1\vec{j} + z_1\vec{k},$

$\vec{V}_2 = \quad x_2\vec{i} + y_2\vec{j} + z_2\vec{k},$

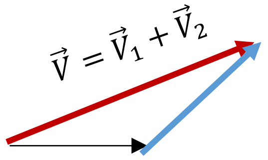
$\vec{V} = \vec{V}_1 + \vec{V}_2 = \quad (x_1 + x_2)\vec{i} + (y_1 + y_2)\vec{j} + (z_1 + z_2)\vec{k}$

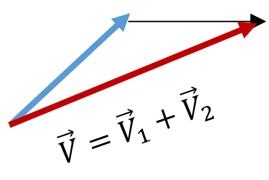






Move the

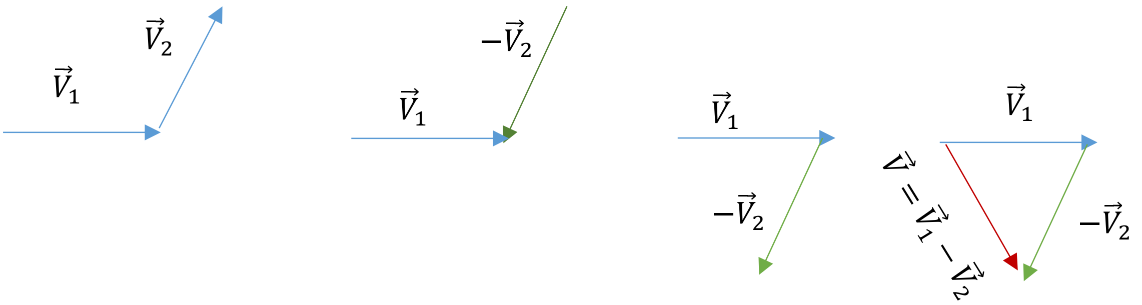




- $\vec{V}_1 + \vec{V}_2 = \vec{V}_2 + \vec{V}_1$
- $\vec{V}_1 + (\vec{V}_2 + \vec{V}_3) = \overrightarrow{(V_1} + \vec{V}_2) + \vec{V}_3$
- $\alpha\vec{V}_1 = \vec{V}_2$, $\vec{V}_2$ has the same direction and magnitude $\|\vec{V}_2\| = \alpha\|\vec{V}_1\|$
- $\alpha(\vec{V}_1 + \vec{V}_2) = \alpha\vec{V}_1 + \alpha\vec{V}_2$

2. Subtraction:

Subtraction is the same operation as addition, except that the subtracted vector must be returned to its negative direction.



$$\vec{V} = \vec{V}_1 - \vec{V}_2 \rightarrow \vec{V} = \vec{V}_1 + (-\vec{V}_2)$$

$\vec{V}_1 = \quad x_1\vec{\imath} + y_1\vec{\jmath} + z_1\vec{k},$

$\vec{V}_2 = \quad x_2\vec{\imath} + y_2\vec{\jmath} + z_2\vec{k},$

$\vec{V} = \vec{V}_1 - \vec{V}_2 = \ (x_1 - x_2)\vec{\imath} + (y_1 - y_2)\vec{\jmath} + (z_1 - z_2)\vec{k}$

3. Product

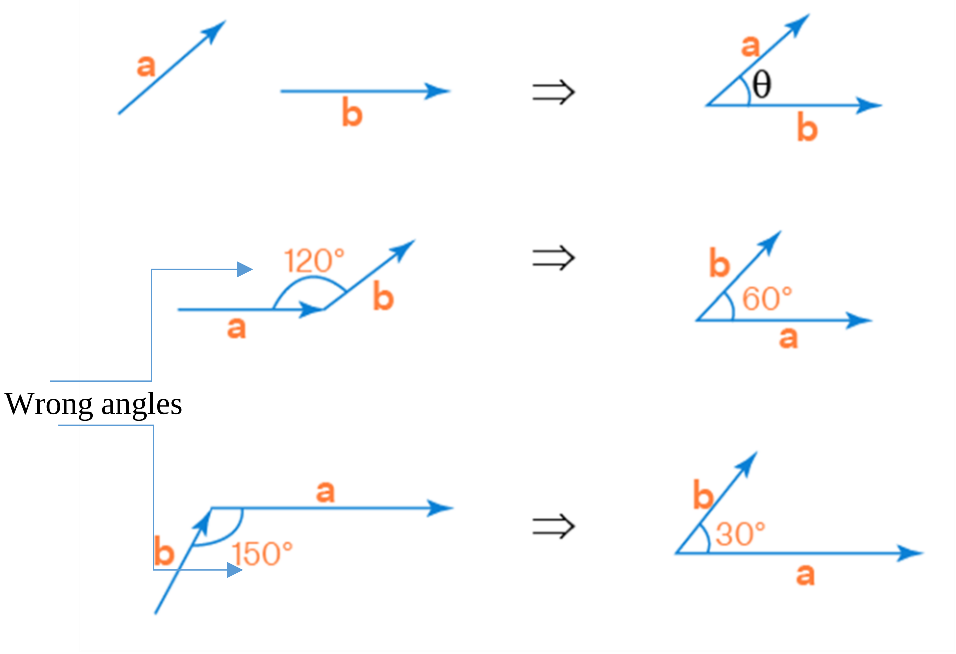
There are two types of product;

4. Scalar product (Dot product)

The result of scalar product of two vectors is a scalar quantity (number), and is defined by this formula,

$$\vec{V}_1 \cdot \vec{V}_2 = \|\vec{V}_1\| \cdot \|\vec{V}_2\| cos(\alpha)$$

α is the **angle between the two vectors**, which is defined as the angle formed at the intersection of their tails (of their beginnings). If the vectors are not joined tail-tail, then we have to join them to find the angle between them.



Properties

- $\vec{V}_1 \cdot \vec{V}_2 = \vec{V}_2 \cdot \vec{V}_1$
- $\vec{V}_1 \cdot (\vec{V}_2 + \vec{V}_3) = \vec{V}_1 \cdot \vec{V}_2 + \vec{V}_1 \cdot \vec{V}_3$
- $\alpha(\overrightarrow{V_1} \cdot \vec{V}_2) = (\alpha\vec{V}_1) \cdot \vec{V}_2 = \vec{V}_1 \cdot (\alpha\vec{V}_2)$
- $\vec{V}_1 \cdot \vec{V}_2 = 0$ only if one of the two is equal to zero or the two vectors are perpendicular.

$$\vec{V}_1 \cdot \vec{V}_2 = x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_1$$

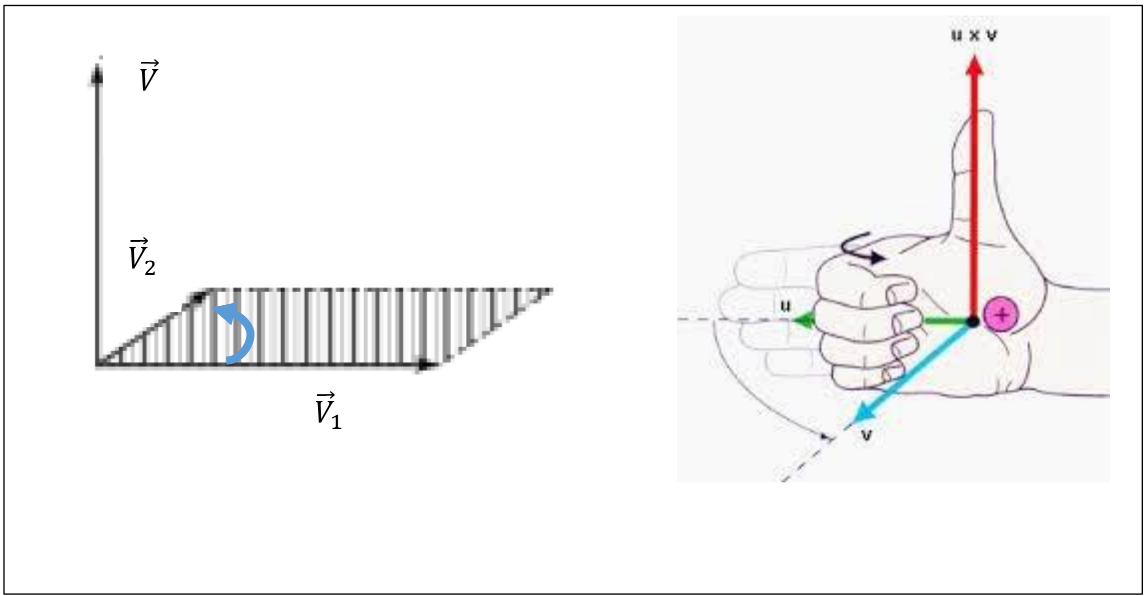
a. Cross product

The result of the cross product of two vectors ($\vec{V}_1$ *and* $\vec{V}_2$) is a third vector $\vec{V}_3$ that is perpendicular to both of them.

$$\vec{V}_3 = \vec{V}_1 \wedge \vec{V}_2$$

The magnitude of the resulting vector is given by

$$\|\vec{V}_3\| = \|\vec{V}_1\| \,.\, \|\vec{V}_2\| \sin(\alpha)$$



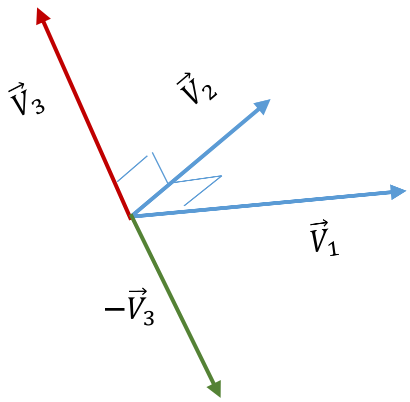
- The module of $\vec{V}_3$ represents the surface constructed by the two vectors $\vec{V}_1$ and $\vec{V}_2$.

$$S = \|\vec{V}_3\| = \|\vec{V}_1\| \,.\, \|\vec{V}_2\| \sin(\alpha)$$

- We conclude the surface of the triangle constructed by these vectors is

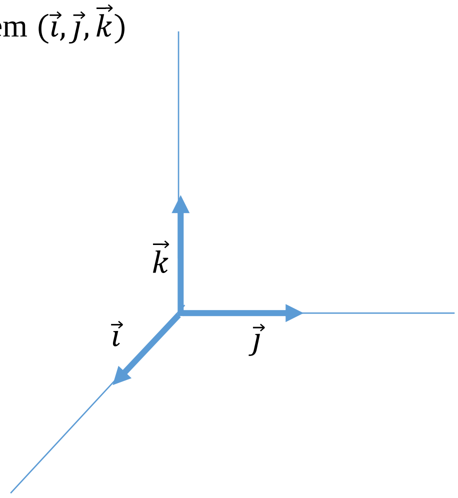
$$S = \frac{1}{2}\|\vec{V}_3\| = \frac{1}{2}\|\vec{V}_1\| \,.\, \|\vec{V}_2\| \sin(\alpha)$$

Properties



- If $\vec{V}_3 = \vec{V}_1 \wedge \vec{V}_2$, then $\vec{V}_2 \wedge \vec{V}_1 = \overline{-V_3}$

- In Cartesian Coordinate System $(\vec{\imath}, \vec{\jmath}, \vec{k})$



$$\vec{i} \wedge \vec{j} = \vec{k}$$
$$\vec{j} \wedge \vec{k} = \vec{i}$$
$$\vec{k} \wedge \vec{i} = \vec{i}$$

$$\vec{i} \wedge \vec{k} = -\vec{j}$$
$$\vec{k} \wedge \vec{j} = -\vec{i}$$
$$\vec{j} \wedge \vec{i} = -\vec{k}$$

- $\vec{V}_1 \wedge (\vec{V}_2 + \vec{V}_3) = \vec{V}_1 \wedge \vec{V}_2 + \vec{V}_1 \wedge \vec{V}_3$
- $\alpha(\overrightarrow{V_1 \wedge V_2}) = (\alpha\vec{V}_1) \wedge \vec{V}_2 = \vec{V}_1 \wedge (\alpha\vec{V}_2)$
- $\vec{V}_1 \wedge \vec{V}_2 = \vec{0}$ only if one of the two is equal to zero or the two vectors are parallels.

➢ The algebraic formula for the vector product is given by

$$\vec{V}_1 \wedge \vec{V}_2 = \begin{vmatrix} \vec{i} & -\vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = \begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix} \vec{i} - \begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix} \vec{j} + \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} \vec{k}$$

$$\vec{V} = \vec{V}_1 \wedge \vec{V}_2 = (y_1 z_2 - z_1 y_2 \vec{\imath} - (x_1 z_2 - z_1 x_2)\vec{\jmath} + (x_1 y_2 - y_1 x_2)\vec{k}$$

➢ **Mixed product**

The mixed product is defined by,

$$\vec{V} = \vec{V}_1 \cdot (\vec{V}_2 \wedge \vec{V}_3)$$

The results of the mixed product is a scalar quantity and represents the volume of the parallelepiped constructed with $\vec{V}_1, \vec{V}_2$ and $\vec{V}_3$.