Exercise 3.1

Determine the domain of the following functions:

$$f_1(x) = \frac{x+1}{1-e^{\frac{1}{x}}} \qquad f_2(x) = \frac{1}{\sqrt{\sin(x)}}$$

$$f_3(x) = e^{\frac{1}{1-x}}\sqrt{x^2-1} \qquad f_4(x) = (1+\ln(x))^{\frac{1}{x}}$$

Exercise 3.2 (*)

Using the definition, prove the following limits:

1. $\lim_{x\to 1} 2x+1 = 3$
2. $\lim_{x\to 2} x^2 = 4$
3. $\lim_{x\to 4} \sqrt{x} = 2$
4. $\lim_{x\to 1} \frac{1}{(1-x)^2} = +\infty$

Exercise 3.3

Calculate the following limits:

1. $\lim_{x\to 0} x\, sin(\frac{1}{x})$
2. $\lim_{x\to +\infty} x\, sin(\frac{1}{x})$
3. $\lim_{x\to 0} \frac{\tan x}{x}$
4. $\lim_{x\to +\infty} \frac{2x^2-3x+1}{x^2+x+1}$
5. $\lim_{x\to 0} \frac{\ln x}{x}$ (*)
6. $\lim_{x\to 0} \frac{\ln x}{x^n}$ (*)
7. $\lim_{x\to +\infty} \frac{\ln x}{x}$ (*)
8. $\lim_{x\to 0} x\, lnx$ (*)
9. $\lim_{x\to +\infty} \frac{e^x}{x}$ (*)
10. $\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a}$, $a > 0$
11. $\lim_{x\to +\infty} x\sqrt{1+\frac{1}{x}}$

Exercise 3.4

Let $a \in R, b \in R$. We define the function f such as:

$$f(x) = \begin{cases} ax+b & if\ x \leq 0 \\ \dfrac{3}{1+x} & if\ x > 0 \end{cases}$$

1. Determine b so that f is continuous on R.
2. Determine a and b so that f is differentiable on R.

Exercise 3.5

Calculate the derivatives of the following functions:

1. $f_1(x) = ln(3 + sin(x))$

* Done in lecture.

2. $f_2\ (x) = \ln \mathrm{x} + \sqrt{1 + \mathrm{x}\ 2}$
3. $f_3\ (x) = \frac{2+cos(x)}{2-cos(x)}$?
4. $f_4(x) = sin\ ((e^x)^2)$

Exercise 3.6

Study the differentiability of the following function and then, calculate the derivative when it exists.

$$f(x)\begin{cases} x^2 + x \ \ if \ x \leq 0 \\ \sin(x)\, if \ 0 < x \leq \pi \\ 1 + \cos x \, if \ x > \pi \end{cases}$$

Exercise 3.7

Using the Hospital formula, calculate the following limits.

1. $\lim\limits_{x \to 0} \frac{1 - cos\ (x)}{e^x - 1}$
2. $\lim\limits_{x \to 0} \frac{x^x - 1}{\ln x - x + 1}$
3. $\lim\limits_{x \to 0} \frac{sin\ (x)}{x^2 - \pi^2}$

Exercise 3.8 (*)

Prove that:

1. $\forall x \in R\ (\tan x)' = 1 + \tan^2 x$
2. $\forall x \in]-1,1[\ (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
3. $\forall x \in]-1,1[\ (\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
4. $\forall x \in R\ (\arctan x)' = \frac{1}{1+x^2}$

Exercise 3.9

Prove that:

1. $\forall x \in R: \cosh x + \sinh x = e^x$
2. $\forall x \in R: \cosh x - \sinh x = e^x$
3. $\forall x \in R: \cosh^2 x - \sinh^2 x = 1$

* Done in lecture.