Exercice 2.1 <u>DONE IN THE LECTURE</u>

(Demonstration of Morgan's Law).

Let A and B be parts of a set E, demonstrate that:

$$1.\ (A \cap B)^c = A^c \cup B^c \quad and \quad 2.\ (A \cup B)^c = A^c \cap B^c$$

Exercise 2.2

1. We consider the following sets:

$$A = \{1, 3, 7, 9, 12\}, B = \{1, 3, 2\}, C = \{3, 4, 7, 9\}, D = \{3, 1\}.$$

Describe the following sets and their cardinals:

$$A \cap B, A\backslash B, A\Delta B, D \times C, B \cap C, C_A D\ (D^c), D \cup A, \mathrm{P}(C).$$

2. Describe the following sets:

$$F = [-2, 1[\cap] - \infty, 0], E = [-2, 1[\cup] - \infty, 0], G = [-2, 1[\Delta] - \infty, 0], H = C_R F$$

Exercise 2.3

Let A, B and C be three subsets of a set E. Prove the following Morgan's laws:

1. (A∩B) ∪ C ⊂ (A∪C) ∩ (B∪C)

Exercise 2.4 HOMEWORK

Let A, B and C be three parts of a set E.

Show that: (A\B) \C = A\ (B ∪ C).

Exercise 2.5

Let $\mathcal{R}$ be the relation defined on R^2 by: x $\mathcal{R}$ y if and only if: $x^2 - y^2 = x - y$

$$(\mathcal{R} = \{(x, y) | x^2 - y^2 = x - y\})$$

1. Show that R is an equivalence relation. (reflexive, symmetric and transitive)
2. find the equivalence class of an element x through the equivalent relation $\mathcal{R}$

Exercise 2.6

Let R be the relation defined on N ∗ by:

$$\mathrm{n}\mathcal{R}\mathrm{m} \iff \exists k \in \mathbb{N}^* : \mathrm{n} = \mathrm{km}$$

$$we\ can\ write\ also:\ \mathcal{R} = \{(n,m)|\mathrm{n} = \mathrm{km}, k \in \mathbb{N}^*\}$$

Show that $\mathcal{R}$ is a partial order relation

Exercise 2.7

In the following exercises, determine whether each equation is a function.

$a.\ 2x + y = -3$ $\quad b.\ y = x^2$ $\quad c.\ x + y^2 = -5$

Exercise 2.8

1. Check whether the following functions are injective, surjective or bijective:

$$f: \mathbb{N} \longrightarrow \mathbb{N}, \quad x \longrightarrow x^2$$

$$g: \mathbb{Z} \longrightarrow \mathbb{N}, \quad x \longrightarrow 4x^2 + 5$$

$$h: \mathbb{R} - \{3\} \longrightarrow \mathbb{R}, \quad x \longrightarrow \frac{2-x}{x+3}$$

2. Calculate h ∘ f and f ∘ h
3. Show that the map u: $]1,\infty[\to]0,\infty[$ defined by: $u\,(x) = \frac{1}{x-1}$ is bijective and calculate its reciprocal function.

Exercise 2.9

Let f : $[0,+\infty[\to [0,+\infty[$ be a function defined by: $f(x) = \left(\sqrt{x}+1\right)^2 - 1$

1. Show that f is bijective.
2. Determine its reciprocal function f^{-1}