University Center of Mila — 2023/2024
Department of Mathematics — M2-S03

Elliptic Curves
Work Sheet 01

Exercise 1. Fix a non-zero homogeneous polynomial

$$F(x,y,z)=\sum_{i=0}^{3}\sum_{j=0}^{3-i} a_{ij}x^{3-i-j}y^iz^j$$

of degree 3 and let C be the plane cubic curve defined by$F=0$.
For $P=[1:0:0]\in\mathbb{P}^2$, show that the following statements hold.

1. $P\in C$ if and only if $a_{00}=0$.
2. P is a singular point of C if and only if $a_{00}=a_{10}=a_{01}=0$.
3. P is a triple point of C if and only if

$$a_{00}=a_{10}=a_{01}=a_{11}=a_{20}=a_{02}=0.$$

Exercise 2. Find the values $a\in\mathbb{C}$ for which the lines of equations

$$ay-z+3ix=0,\quad -iax+y-iz=0,\quad 3iz+5x+y=0$$

of $\mathbb{P}^2(\mathbb{C})$ are concurrent.

Exercise 3. Show that the points

$$[1,2,2],\quad [3,1,4],\quad [2,-1,2]$$

of the real projective plane are collinear, and find an equation of the line containing them.