Democratic and Popular Republic of Algeria

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Exercises Serie N° 1

Exercise 1

Let A and B be two non-empty and bounded sets. we define:

$-A = \{-x \mid x \in A\}$, $A + B = \{x = a + b \mid a \in A, b \in B\}$ *and* $A - B = \{x = a - b \mid a \in A, b \in B\}$

1. *Show that:* $\sup(-A) = -\inf(A)$ *and* $\inf(-A) = -\sup(A)$.
2. *Show that if for all* $a \in A$ *and* $b \in B$ *we have* $a \leq b$, *then* $\sup(A) \leq \inf(B)$.
3. *Show that* $A \cup B$ *is a bounded subset of* $\mathbb{R}$ *and:*
   (a) $\sup(A \cup B) = \max(\sup(A), \sup(B))$.
   (b) $\inf(A \cup B) = \min(\inf(A), \inf(B))$. *(*)*
4. *Show that* $\sup(A) + \sup(B)$ *is an upper bound of* $A + B$ *and:*
   (a) $\sup(A + B) = \sup(A) + \sup(B)$.
   (b) $\inf(A + B) = \inf(A) + \inf(B)$.

Exercise 2

1. *Show that if* $r \in \mathbb{Q}$ *and* $x \notin \mathbb{Q}$ *then* $r + x \notin \mathbb{Q}$ *and if* $r \neq 0$ *then* $r.x \notin \mathbb{Q}$.
2. *Show that* $\sqrt{2} \notin \mathbb{Q}$.
3. *Show that* $\dfrac{ln3}{ln2}$ *is irrational.*
4. *Let a and b be two positive rationals such that* $\sqrt{a}$ *and* $\sqrt{b}$ *are irrational. show that* $\sqrt{a} + \sqrt{b}$ *is irrational. (*)*

Exercise 3

Let A and B be two subsets of $\mathbb{R}$ *such that* $B \subset A$. *Show that:*

1. *A is bounded* $\Longrightarrow$ *B is bounded .*
2. $\inf(A) \leq \inf(B)$, *and* $\sup(A) \geq \sup(B)$.

Exercise 4

Let $A = \{a_n \in \mathbb{R} \mid a_n = \dfrac{n+3}{\frac{n}{4}+1};\ n \in \mathbb{N}\}$ *and* $B = \{b_n \in \mathbb{R} \mid b_n = \frac{1}{n^2} + \dfrac{2}{n} + 4;\ n \in \mathbb{N}^*\}$.

1. *Show that A and B are bounded in* $\mathbb{R}$ *and that* $\sup(A) = \inf(B)$.
2. *Determine* $\sup(A)$ *and* $\inf(B)$.

Exercise 5

Determine the supremum (the upper bound) and infimum (the lower bound), if they exist of the following sets:
$A = \{ax + b \mid x \in [-2, 1] \quad and \quad a, b \in \mathbb{R}\}$, $B = \{2 - \frac{1}{n};\ n \in \mathbb{N}\}$; $C = \{\sin \frac{2n\Pi}{7};\ n \in \mathbb{Z}\}$. *(*)*

Exercise 6

1. *Write the following numbers in the form* $a + ib, (a,\ b \in \mathbb{R})$*:*

   $z_1 = \dfrac{5+2i}{1-2i}$, $z_2 = -\dfrac{2}{1-i\sqrt{3}}$, $z_3 = \dfrac{2+5i}{1-i} + \dfrac{2-5i}{1+i}$ *(*)*

2. *Solve the following equations:*

   $z^4 + (3-6i)z^2 - 8 - 6i = 0,\ z^2 - \sqrt{3}z - i = 0.$*(*)*

Exercise 7

Using complex numbers, calculate $\cos(5\theta)$ *and* $\sin(5\theta)$ *in terms of* $\cos(\theta)$ *and* $\sin(\theta)$.

Exercise 8

1. *Calculate the modulus and the argument of* $u = \dfrac{\sqrt{6} - i\sqrt{2}}{2}$ *and* $v = 1 - i$.
2. *Deduce the modulus and the argement of* $\dfrac{u}{v}$.

Exercise 9 *(Supplementary)*

Let z *be an* n^{th} *root of -1, so* $z^n = -1$ *with* $n > 2$ *and* $z \neq -1$

calculate $S_n = \sum_{k=0}^{n-1} Z^{2k} = 1 + z^2 + Z^4 + ... + z^{2(n-1)}$.

Exercises marked with (*) are left to students.