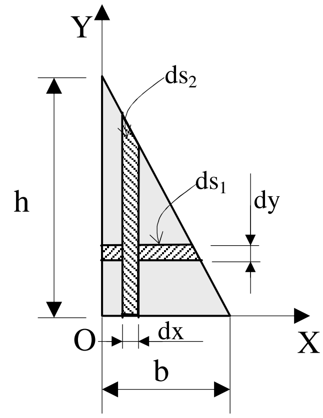
Correction TD3 : Caractéristiques géométriques des sections

Exercice 1

Calcul des coordonnées X_G et Y_G des centres de gravité des sections de la figure 1.

Section1 :

$$X_G = \frac{\iint_S x.dS_2}{S} \text{ ,or on a : } dS_2 = \frac{h(b-x)}{b}.dx \text{ alors :}$$

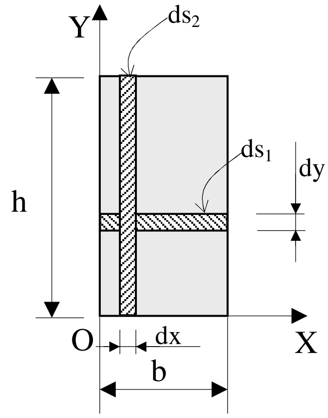


$$X_G = \frac{\int_0^b x\frac{h(b-x)}{b}.dx}{\frac{bh}{2}} = \int_0^b \frac{2(bhx-hx^2)dx}{b^2h} = \frac{2\left[\frac{hbx^2}{2}-\frac{hx^3}{3}\right]_0^b}{b^2h} = \frac{b}{3}$$

$$Y_G = \frac{\iint_S y.dS_1}{S} \text{ or on a : } dS_1 = \frac{b(h-y)}{h}.dy \text{ alors :}$$

$$Y_G = \frac{\int_0^h y\frac{b(h-y)}{h}.dy}{\frac{bh}{2}} = \int_0^h \frac{2(bhy-by^2)dy}{b^2h} = \frac{2\left[\frac{hby^2}{2}-\frac{by^3}{3}\right]_0^h}{b^2h} = \frac{h}{3}$$

Section2 :



$$X_G = \frac{\iint_S x.dS_2}{S} \text{ ,or on a : } dS_2 = h.dx \text{ alors : } X_G = \frac{\int_0^b h.x.dx}{bh} = \frac{\left[\frac{x^2}{2}\right]_0^b}{b} = \frac{b}{2}$$

$$Y_G = \frac{\iint_S y.dS_1}{S} \text{ , or on a : } dS_1 = b.dx \text{ alors : } Y_G = \frac{\int_0^h b.y.dy}{bh} = \frac{\left[\frac{y^2}{2}\right]_0^h}{h} = \frac{h}{2}$$

Exercice 2

Calcul des coordonnées X_G et Y_G du centre de gravité de la section de la figure 2.

On décompose la section en 3 sections simples (rectangles) :

$$X_G = \frac{\sum_1^n X_{Gi}\cdot S_i}{S} \quad \text{et} \quad Y_G = \frac{\sum_1^n Y_{Gi}\cdot S_i}{S}$$

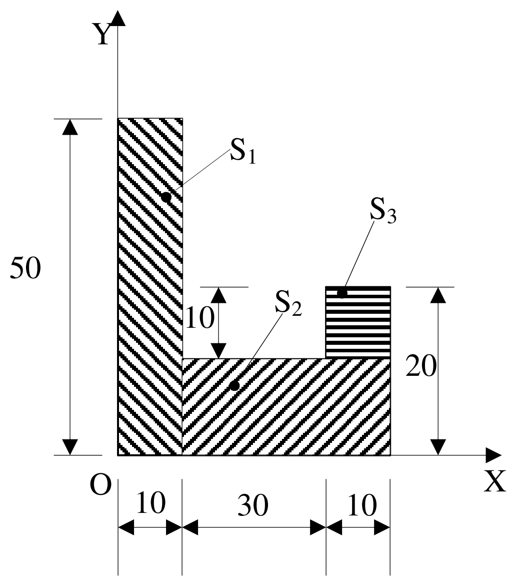


Figure 2

N°	b	h	S_i	X_i	Y_i	$X_i.\ S_i$	$Y_i.\ S_i$
1	10	50	500	5	25	2500	12500
2	40	10	400	30	5	12000	2000
3	10	~~20~~	~~200~~	45	~~10~~	~~9000~~	~~2000~~
Σ			~~1100~~			~~23500~~	~~16500~~

$X_G = \frac{23500}{1100} = 21{,}3\,cm$ ~~et~~ $Y_G = \frac{16500}{1100} = 15\,cm$

EXERCICE3

1) Calcul des moments quadratiques $I_{GX'}$ et $I_{GY'}$ de la section de la figure 3.

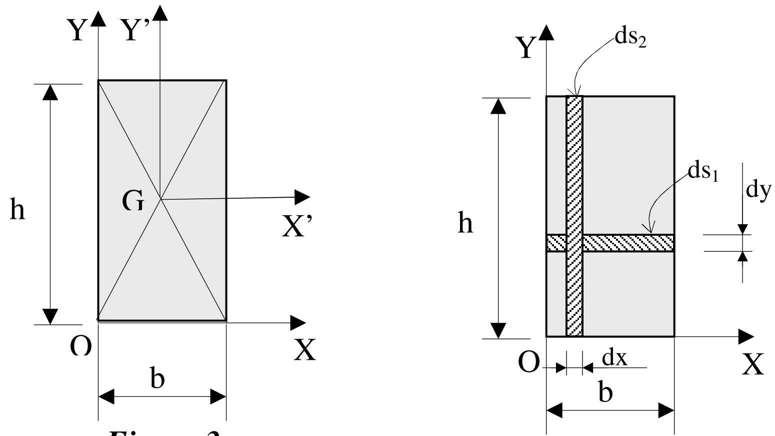


Figure 3

$$I_{GX'} = \iint_S y^2.dS \text{ , or on a : } dS_1 = b.dy \text{ alors : } I_{GX'} = \int_{-h/2}^{h/2} b.y^2.dy = b.\left[\frac{y^3}{3}\right]_{-h/2}^{h/2} = \frac{b.h^3}{12}$$

$$I_{GY'} = \iint_S x^2.dS \text{ , or on a : } dS_2 = h.dx \text{ alors : } I_{GX'} = \int_{-b/2}^{b/2} h.x^2.dx = h.\left[\frac{x^3}{3}\right]_{-b/2}^{b/2} = \frac{h.b^3}{12}$$

2) Calcul des moments quadratiques I_{OX} et I_{OY} de cette section.

On applique le théorème de Huygens :

$$I_{Ox} = I_{Gx'} + S.Y_G^2 = \frac{b.h^3}{12} + b.h.\left[\frac{h}{2}\right]^2 = \frac{b.h^3}{3}$$

$$I_{Oy} = I_{Gy'} + S.X_G^2 = \frac{h.b^3}{12} + b.h.\left[\frac{b}{2}\right]^2 = \frac{h.b^3}{3}$$

Exercice4

Calcul des moments quadratiques I_{OX} et I_{OY} de la section de la figure 2.

$$I_{Ox} = \sum_{1}^{n} I_{Oxi} \quad \text{et} \quad I_{Oy} = \sum_{1}^{n} I_{Oyi}$$

Avec $I_{Oxi} = I_{Gxi} + S_i \cdot Y_{Gi}^2$ et $I_{Oyi} = I_{Gyi} + S_i \cdot X_{Gi}^2$

N°	b	h	Si	IG_{Xi}	IG_{Yi}	XGi	YGi	IOXi	IOYi
1	10	50	500	104166.67	4166.67	5	25	416666.67	16666.67
2	40	10	400	3333.33	53333.33	30	5	13333.33	413333.33
3	10	~~20~~ → 10	~~200~~ → 100	~~6666.67~~ → 833,33	1666.67 → 833,33	45	15	~~51666.67~~ → 23333,33	~~406666.67~~ → 203333,33
Σ								~~481666.67~~	~~836666.67~~

$I_{Ox} =$ ~~114166,7~~ cm^4 → 453333,33 et $I_{Oy} =$ ~~59166,7~~ cm^4 → 633333,33