**Solution de la série n° :02**

**Exercice 01 :**

01/

Les points d'interpolation :$x\_{0}=0,x\_{1}=\frac{1}{6},x\_{2}=\frac{1}{2}$

$$f\left(x\right)=\sin((⫪x))$$

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |  |
| --- | --- | --- | --- |
| $$ x\_{i}$$ | 0 | 1/6 | ½ |
| $$ y\_{i}$$ | 0 | 1/6 | 1 |

 |  |  |  |
| xi |  |  |  |
| $$ p\_{2}\left(x\right)=\sum\_{k=0}^{2}f\left(x\_{k}\right).l\_{k}(x) $$ |  |  |  |

$$L\_{20}=\frac{(x-x\_{1})(x-x\_{2})}{(x\_{0}-x\_{1})(x\_{0}-x\_{2})}=\frac{(x-1/6)(x-1/2)}{(0-1/6)(0-1/2)}=\frac{x^{2}-2/3x+1/12}{1/12}$$

$$L\_{21}=\frac{(x-x\_{0})(x-x\_{2})}{(x\_{1}-x\_{0})(x\_{1}-x\_{2})}=\frac{(x-0)(x-1/2)}{(1/6-0)(1/6-1/2)}=\frac{x^{2}-1/2x}{-1/18}$$

$$L\_{22}=\frac{(x-x\_{0})(x-x\_{1})}{(x\_{2}-x\_{0})(x\_{2}-x\_{1})}=\frac{(x-0)(x-1/6)}{(1/2-0)(1/2-1/6)}=\frac{x^{2}-1/6x}{1/6}$$

$$=\frac{x^{2}-2/3x+1/12}{1/12}.0+\frac{x^{2}-1/2x}{-1/18}.\frac{1}{2}+\frac{x^{2}-1/6x×1}{1/2}$$

$$$$

2/ la valeur exacte de l'erreur : $x=\frac{1}{8}$

$$\left|p\_{2}\left(x\right)-f\left(x\right)\right|=\left|p\_{2}\left(\frac{1}{8}\right)-f\left(\frac{1}{8}\right)\right|=\left|3\left(\frac{1}{8}\right)^{2}+\frac{7}{2}.\frac{1}{8}-\sin(\frac{⫪}{8})\right|$$

 =

**Exercice 02 :**

|  |  |  |  |
| --- | --- | --- | --- |
| $$x\_{i}$$ | 2 | 2.5 | 4 |
| $$f(x\_{i})$$ | 0.5 | 0.4 | 0.25 |

$$L\_{20}=\frac{(x-x\_{1})(x-x\_{2})}{(x\_{0}-x\_{1})(x\_{0}-x\_{2})}=\frac{(x-0.5)(x-4)}{(2-2.5)(2-4)}=x^{2}-6.5x+10$$

$$L\_{21}=\frac{(x-x\_{0})(x-x\_{2})}{(x\_{1}-x\_{0})(x\_{1}-x\_{2})}=\frac{(x-2)(x-4)}{(2.5-2)(2.5-4)}=\frac{x^{2}-6x+8}{-0.75}$$

$$L\_{22}=\frac{(x-x\_{0})(x-x\_{1})}{(x\_{2}-x\_{0})(x\_{2}-x\_{0})}=\frac{(x-2)(x-0.5)}{(4-2.5)(4-2)}=\frac{x^{2}-4.5x+5}{3}$$

$$p\_{2}(x) =\sum\_{k=0}^{2}f\left(x\_{k}\right).l\_{k}(x)$$

=$L\_{20}.f\left(x\_{0}\right)+L\_{21}.f\left(x\_{1}\right)+L\_{22}.f\left(x\_{2}\right)$

$$ =\left(x^{2}-6.5x+10\right).0.5+\left(\frac{x^{2}-6x+8}{-0.75}\right).0.4+\left(\frac{x^{2}-4.5x+5}{3}\right).0.25$$

= (0.5-$\frac{0.4}{0.75}+\frac{0.25}{3})x^{2}+\left(\frac{-6.5}{2}+\frac{2.4}{0.75}-\frac{4.25}{12}\right)x+(5-\frac{3.2}{0.75}+\frac{5}{12})$

- La valeur de f(3)

$$f\left(3\right)=0.05(3)^{2}-0425\left(3\right)+1.15=0.325$$

- La majoration de l'erreur (Méthode de Lagrange)

$$R\_{n}(x)\leq \left|\frac{1}{\left(n+1\right)!}\prod\_{i=0}^{n}\left(x-x\_{i}\right)f^{\left(n+1\right)}(ξ)\right|$$

$$f\left(x\right)=\frac{1}{x}⇒f^{'}\left(x\right)=\frac{1}{x^{2}}⇒f^{''}\left(x\right)=\frac{2}{x^{3}}⇒f^{'''}\left(x\right)=\frac{-6}{x^{4}}$$

$$ξ=2⇒f^{'''}\left(2\right)=\frac{-6}{2^{4}}=\frac{6}{16}=\frac{3}{8}$$

$$R\_{2}(x)\leq \frac{1}{6}\left|x-2\right|\left|x-2.5\right|\left|x-4\right|\frac{3}{8}$$

$$$$

$$f\left(3\right)=\frac{1}{3}$$

**L'erreur exacte** : $\left|p\_{2}(3)-f(3)\right|=\left|0.325-\frac{1}{3}\right|=0.10^{-3}$

\* $f\left(x\right)=\frac{1}{x}⇒f^{'}\left(x\right)=\frac{-1}{x^{2}}$

$$⇒f^{'}\left(3\right)=-\frac{1}{9}=-0.111$$

$$p\_{2}\left(x\right)=0.05x^{2}-0.425x+1.15$$

$$p\_{2}^{'}\left(x\right)0.1x-0.425$$

$$p\_{2}^{'}\left(3\right)=0.1\left(3\right)-0.425=-0.125$$

L'erreur exacte :$\left|-0.125-(-0.111)\right|=0.014$

**Exercice 03 :**

Le polynôme d'interpolation de Newton :

$$p\_{2}\left(x\right)=y\_{0}+∆y\_{0}\left(x-x\_{0}\right)+∆^{2}y\_{0}\left(x-x\_{0}\right)\left(x-x\_{1}\right)+∆^{3}y\_{0}\left(x-x\_{0}\right)\left(x-x\_{1}\right)\left(x-x\_{2}\right)+\cdots \cdots \cdots +∆^{n}y\_{0}\left(x-x\_{0}\right)\left(x-x\_{1}\right)\cdots \left(x-x\_{n-1}\right)$$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x\_{i}$$ | $$y\_{i}$$ | $$∆y\_{i}$$ | $$∆^{2}y\_{i}$$ | $$∆^{3}y\_{i}$$ |
| 0124 | 1325 | 213 | $$\frac{-3}{2}$$2 | $$\frac{7}{6}$$ |

4 points d'interp$⇒$le polyn de degre3

$$p\_{3}\left(x\right)=1+\left(x-1\right)×2+\left(\frac{-3}{2}\right)\left(x-1\right)\left(x-2\right)+\frac{7}{6}(x-1)(x-2)(x-3)$$

$$$$

**Exercice 04 :**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x\_{i}$$ | $$y\_{i}$$ | $$∆y\_{i}$$ | $$∆^{2}y\_{i}$$ | $$∆^{3}y\_{i}$$ |
| 0124 | 13981 | 2636 | 210 | 2 |

$$∆y\_{0}=\frac{y\_{1}-y\_{0}}{x\_{1}-x\_{0}}.∆y\_{1}=\frac{y\_{2}-y\_{1}}{x\_{2}-x\_{1}}$$

$$∆^{2}y\_{0}=\frac{∆y\_{1}-∆y\_{0}}{x\_{1}-x\_{0}}$$

$$.∆^{2}y\_{1}=\frac{∆^{2}y\_{2}-∆^{2}y\_{1}}{x\_{3}-x\_{1}}$$

$$∆^{3}y\_{0}=\frac{∆^{2}y\_{1}-∆^{2}y\_{0}}{x\_{3}-x\_{0}}$$

$$ p\_{3}\left(x\right)=1+2x+2x\left(X-1\right)\left(x-1\right)+2\left(2x^{2}-2x\right)(x-2)$$

$$$$

$$p\_{3}\left(3\right)=2(3)^{3}-4\left(3^{2}\right)+4\left(3\right)+1=54-36+13=31$$

$$p\_{3}\left(3\right)=31$$