University Center Abdelhafid Boussouf - Mila | University year 2022-2023
Institute : Sciences and Technologie | Departement: of Mathematics and computer sc
Discrete Dynamical Systems | Master I
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Exercises [One-Dimensional Maps]

Exercise 1

Hyperbolic Fixed Points

1. Consider the map $f_\lambda(x) = 1 - \lambda x^2$ defined on the interval$[-1, 1]$, where $\lambda \in (0, 2]$. Find the fixed points of $f_\lambda(x)$ and determine their stability

2. Consider the logistic map $L_\mu(x) = \mu x(1 - x)$, $x \in [0, 1]$
.
$$L_\mu(x) : [0, 1] \to [0, 1], 0 < \mu \leqslant 4$$
Find all fixed point of the map $L_\mu(x)$ and determine their stability.

   Nonhyperbolic Fixed Points

3. Consider the map $f(x) = -x^3 + x$
   Find the fixed points of $f(x)$ and determine their stability

4. Consider the map $f(x) = x^2 + 3x$ on the interval $[-3, 3]$ Find the equilibrium points and then determine their stability.

Exercise 2

Periodic Points and their Stability

1. Consider the map $f(x) = |x-3| \quad x|inR$. Find all fixed points and all the $2-$periodic points,

2. Consider the map
$$f(x) = \begin{cases} 2x \text{ si } 0 < x < 0.5 \\ 2(1-x) \text{ si } 0.5 \leq x \leq 1 \end{cases}.$$
. Find all $2-$periodic points and Find all $3-$periodic points

3. Consider the difference equation$x(n+1) = f(x(n))$ where $f(x) = 1 - x^2$ is defined on the interval $[-1, 1]$. Find all the $2-$periodic cycles, $3-$periodic cycles, and $4-$periodic cycles of the difference equation and determine their stability.

4. Find the period of the point $\frac{1}{8}(5+\sqrt{5})$for the map $f(x) = 4x(1-x), x \in [0,1]$) Also determine its stability

Exercise 3

Attraction and Bifurcation

1. Find the fixed points of the one-dimensional map $f(x) = x + sin(x), x \in R$. Also find the basins of attraction.

2. Consider the family of maps $L_\mu(x) = \mu x(1-x)$, $x \in [0,1]$,

   1.Find all fixed points

   2.Carry out the bifurcation analysis

   3. Draw the bifurcation diagram.

3. Consider the family of maps $f_c(x) = c - x^2$, where c is scalar

   1.Find all fixed points

   2.Carry out the bifurcation analysis

   3. Draw the bifurcation diagram.