University Center Abdelhafid Boussouf - Mila   University year 2022-2023

Institute : Sciences and Technologie   Departement: of Mathematics and computer s

Discrete Dynamical Systems   Master I

Dr.Khaoula Rouibah   r.khoula@centre-univ-mila.dz

Exercises

Exercise 1

(a)   the following maps find the compositions $f^2(x)$ and $f^3(x)$:

(i) $f(x) = x^2 - 5$ **(ii)** $f(x) = \sqrt{x+2}$ **(iii)** $f(x) = 3^x$ **(iv)** $f(x) = rx(1-x), x \in [0,1]$ **(v)** $f(x) = x + x^3$ **(vi)** $f(x) = \sin x$ **(vii)** $f(x) = \tan^{-1}(x)$ **(viii)** $f(x) = -\frac{3}{2}x^2 + \frac{5}{2}x + 1$

(b)   Consider the map $x(k+1) = ax(k)$ with $x(0) = b, a$ and b are given constants. Find an iterative formula for $x(k)$.

(c)   Find compositions of the maps given below

(i) $f(x) = x, g(x) = x^2$;

(ii) $f(x) = x^3 - x, g(x) = x^2$

Exercise 2

(a) Define the fixed point in a continuous dynamic system and in discrete dynamic system and concluded the relationship between them

(b)   What do you mean by a fixed point of a map $f : \mathbb{R}^n \to \mathbb{R}^n$? Give geometrical interpretation of fixed point of a map. How do you find fixed points graphically?

(c) How do you explain the fixed point of a map in the context of flow in a discrete system?

Exercise 3

Consider the map $f(x) = x^2 + k, x \in \mathbb{R}$. Find the values of k for which the map f has

(i) two fixed points,

(ii) only one fixed point,

(iii) no fixed points.

Exercise 4

find the solution of the difference equation $f(x) = ax$

1. Find all fixed and eventually fixed points of the map $F_\mu(x) = \mu x(1 - x)$.

Find the value of μ that makes the logistic equation becomes a dynamic system

1. Find all fixed and eventually fixed points of the map $f(x) = |x - 1|$.

2. Consider the logistic map $F_\mu(x) = \mu x(1 - x)$.

(a) Draw the cobweb diagram for $\mu = 2, 2.5, 3.2$.

(b) Determine the stability of the equilibrium points for the values of μ in part (a).

3. (a) Find a function with four fixed points, all of which are unstable.

(b) Find a function with no fixed points.

(c) Find a function with a stable and an unstable fixed point.

4. Find the equilibrium points and determine their stability for the map

$f(x) = 5 - \frac{6}{x}$.