correction interrogation 30mn

$$\begin{cases} \dot{x} = \begin{bmatrix} -2 & 1 \\ 0 & 4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 0 & 1 \end{bmatrix} x. \end{cases}$$

1°- stabilité

$$D(s) = \det(sI - A) = (s+2)(s-4)$$

$\longrightarrow s_{1,2} = -2 ; 4 \longrightarrow$ système instable.

2°- FT.

$$H(s) = C(sI - A)^{-1} B = \frac{s+2}{s^2 - 2s - 8} \equiv \frac{1}{s-4}$$

3°- $y(t)$

$$Y(s) = C X(s) = C\left(\phi(s) x(0) + \phi(s) B U(s)\right)$$

$$= C.\phi(s)\left(x(0) + B U(s)\right).$$

$$\phi(s) = (sI - A)^{-1} = \frac{1}{(s-4)(s+2)} \begin{bmatrix} s+4 & 1 \\ 0 & s-2 \end{bmatrix}$$

$$Y(s) = \frac{1}{(s-4)(s+2)} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s-4 & 1 \\ 0 & s+2 \end{bmatrix} \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{1}{s} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$= \frac{s+1}{s(s-4)} = \frac{-1/4}{s} + \frac{5/4}{s-4}$$

$$y(t) = \mathcal{L}^{-1}\left[Y(s)\right] = \mathcal{L}^{-1}\left[\frac{s+1}{s(s-4)}\right]$$

$$= -\frac{1}{4} + \frac{5}{4} e^{4t}$$

4°- commandabilité :

$$M_c = [B \quad AB] = \begin{bmatrix} 0 & 1 \\ 1 & 4 \end{bmatrix} ; \det(M_c) = -1 \neq 0$$

$\longrightarrow$ système commandable.

5°- Forme canonique de commande.

$\bar{x} = P x$ changement de Base.

$$\rightarrow \begin{cases} \dot{\bar{x}} = \bar{A} x + \bar{B} u \\ \bar{y} = \bar{C} x \end{cases}$$ forme cannonique

tel que $\bar{A} = P^{-1} A P$; $\bar{B} = P^{-1} B$ et $\bar{C} = C P$

$P = ?$

on a : $M_c = \begin{bmatrix} 0 & 1 \\ 1 & 4 \end{bmatrix} \longrightarrow M_c^{-1} = \begin{bmatrix} 4 & -1 \\ -1 & 0 \end{bmatrix}$

$$t_2 = [0 \quad 1] M_c^{-1} = [1 \quad 0].$$

$$P^{-1} = \begin{bmatrix} t_2 \\ t_2 . A \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \longrightarrow P = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 0 & 1 \\ 8 & 2 \end{bmatrix} ; \bar{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} ; \bar{C} = [2 \quad 1]$$

6°- $u \stackrel{?}{=} Kx + y_r$, $K = ?$

$$D(s) \equiv P_{BO}(s) = (s+2)(s-4) = s^2 - 2s - 8$$

$$P_{dés}(s) = (s+3)(s+8) = s^2 + 9s + 18$$

$$\longrightarrow \begin{cases} \bar{K}_1 = 18 - (-8) = 26 \\ \bar{K}_2 = 9 - (-2) = 11 \end{cases}$$

$\longrightarrow \bar{K} = [26 \quad 11].$

$K = \bar{K} P^{-1} = [4 \quad 11].$

7°- $y(\infty) = ?$

$$y(\infty) = \lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s)$$

$$= \lim_{s \to 0} s H_{BF}(s) U(s) = \lim_{s \to 0} s H_{BF}(s) \cdot \frac{1}{s}$$

$$= \lim_{s \to 0} H_{BF}(s) = H_{BF}(0)$$

$$= -C(A - BK)^{-1} \cdot B$$

$$= 0{,}11$$