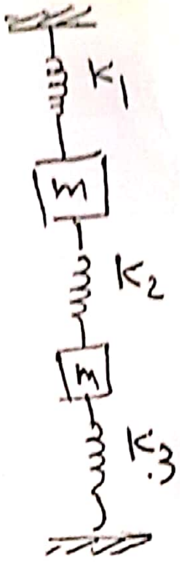
Vibrations : Serie N°4

Ex : 1



$\mathcal{L} = T - U$

$m_1 = m_2 = m$

$K_1 = K_2 = K_3 = K$

$T = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2$

$U = \frac{1}{2} K x_1^2 + \frac{1}{2} K x_2^2 + \frac{1}{2} K (x_2 - x_1)^2$

$\mathcal{L} = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2 - \frac{1}{2} K x_1^2 - \frac{1}{2} K x_2^2 - \frac{1}{2} K (x_2 - x_1)^2$

Le système → libre non amorti.

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_1}\right) - \frac{\partial \mathcal{L}}{\partial x_1} = 0 \\ \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_2}\right) - \frac{\partial \mathcal{L}}{\partial x_2} = 0 \end{cases} \Rightarrow$$

$$\begin{cases} \cancel{m\ddot{x}_1 + 2Kx_1} - Kx_2 = 0 \\ \cancel{m\ddot{x}_2} \end{cases}$$

$$\begin{cases} m\ddot{x}_1 + 2Kx_1 - Kx_2 = 0 \\ m\ddot{x}_2 + 2Kx_2 + Kx_1 = 0 \end{cases} \Rightarrow$$

Supposons que la solution du système et la somme des oscillation harmonique.

$x_1 = A_1 \cos(\omega t + \varphi)$; $x_2 = A_2 \cos(\omega t + \varphi)$

$\dot{x}_1 = -A_1\omega \sin(\omega t + \varphi)$; $\dot{x}_2 = -A_2 \omega \sin(\omega t + \varphi)$

$\ddot{x}_2 = -A_1 \omega^2 \cos(\omega t + \varphi)$; $\ddot{x}_2 = -A_2 \omega^2 \cos(\omega t + \varphi)$

Par remplacement,

$$\begin{bmatrix} (-m\omega^2 + 2K) & -K \\ -K & (-m\omega^2 + 2K) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$$

$A_1 \neq 0$, $A_2 \neq 0$

$m^2\omega^4 - 4Km\omega^2 + 3K^2 = 0$

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$$\frac{\partial L}{\partial \dot\varphi} = ml^2\dot\varphi + ml^2\dot\theta \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\varphi}\right) = ml^2\ddot\varphi + ml^2\ddot\theta$$

$$\frac{\partial L}{\partial \varphi} = -mgl\varphi \Rightarrow$$

$$ml^2\ddot\varphi + ml^2\ddot\theta + mgl\varphi = 0 \quad \text{— ②}$$

$$\begin{cases} 2ml^2\ddot\theta + ml^2\ddot\varphi + 2mgl\theta = 0 \\ ml^2\ddot\theta + ml^2\ddot\varphi + mgl\varphi = 0 \end{cases}$$

$$\begin{cases} 2l\ddot\theta + l\ddot\varphi + 2g\theta = 0 \\ l\ddot\theta + l\ddot\varphi + g\varphi = 0 \end{cases}$$

$$\begin{cases} \theta = A_1 \cos(\omega t + \varphi) \\ \varphi = A_2 \cos(\omega t + \varphi) \end{cases}$$

$$\begin{cases} \ddot\theta = -\omega^2 A_1 \cos(\omega t + \varphi) \\ \ddot\varphi = -\omega^2 A_2 \cos(\omega t + \varphi) \end{cases}$$

بالتعويض.

$$\begin{bmatrix} -2l\omega^2 + 2g & -l\omega^2 \\ -l\omega^2 & -l\omega^2 + g \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$$

محدد المصفوفة الأولى يجب أن يكون معدوما.

$$(-2l\omega^2 + 2g)(-l\omega^2 + g) - l^2\omega^4 = 0$$

$$2l^2\omega^4 - 2l\omega^2 g - 2lg\omega^2 + 2g^2 - l^2\omega^4 = 0$$

$$l^2\omega^4 - 4lg\omega^2 + 2g^2 = 0$$

$$\omega^4 - 4\frac{g}{l}\omega^2 + 2\frac{g^2}{l^2} = 0$$

$$\Rightarrow \omega_1^2 = (2 - \sqrt{2})\,\omega_n^2$$

تردد نواس بسيط

$$\begin{cases} \omega_1^2 = (2 - \sqrt{2})\frac{g}{l} \\ \omega_2^2 = (2 + \sqrt{2})\frac{g}{l} \end{cases}$$

[illegible]

$k_1 = k_f = 18$ KN/m

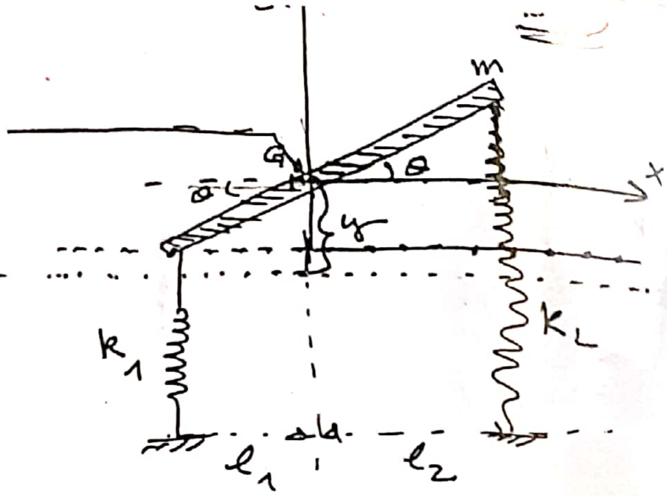
$K_2 = k_r = 28$ KN/m

$l_1 = 1{,}0$ m

$l_2 = 1{,}5$ m

Exercice 03 :

Centre de gravité



l'énergie cinétique et potentiel

$$T = \frac{1}{2} m \dot{y}^2 + \frac{1}{2} I_G \dot{\theta}^2$$

$$U = \frac{1}{2} K_1 (y - l_1\theta)^2 + \frac{1}{2} K_2 (y + l_2\theta)^2$$

$$\mathcal{L} = T - U \Rightarrow$$

$$\boxed{\mathcal{L} = \frac{1}{2} m \dot{y}^2 + \frac{1}{2} I_G \dot{\theta}^2 - \frac{1}{2} K_1 (y - l_1\theta)^2 - \frac{1}{2} K_2 (y + l_2\theta)^2}$$

1) $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}}\right) = m\ddot{y}$; $\frac{\partial \mathcal{L}}{\partial y} = -K_1(y - l_1\theta) - K_2(y + l_2\theta)$

$$= -(K_1 + K_2) y + (K_1 l_1 - K_2 l_2)\theta$$

$$m\ddot{y} + (K_1 + K_2) y - (K_1 l_1 - K_2 l_2)\theta = 0 \quad \text{— (1)}$$

2) $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) = I_G \ddot{\theta}$; $\frac{\partial \mathcal{L}}{\partial \theta} = K_1 l_1 (y - l_1\theta) - K_2 l_2 (y + l_2\theta)$

$$= (K_1 l_1 - K_2 l_2) y + (K_1 l_1^2 + K_2 l_2^2)\theta$$

$$I_G \ddot{\theta} + (K_1 l_1^2 + K_2 l_2^2)\theta - (K_1 l_1 - K_2 l_2) y = 0$$

$$\begin{cases} m\ddot{y} + (K_1 + K_2) y - (K_1 l_1 - K_2 l_2)\theta = 0 & \text{— 1-a} \\ I_G \ddot{\theta} + (K_1 l_1^2 + K_2 l_2^2)\theta - (K_1 l_1 - K_2 l_2) y = 0 & \text{1-b} \end{cases}$$

La solution harmonique doit être sous la forme.

$$x(t) = X \cos(\omega t + \varphi)$$

$$\theta(t) = \Theta \cos(\omega t + \varphi)$$

Ex 3) $\frac{1}{2}$

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Par dérivation et remplacement on trouve

$$\begin{bmatrix} -m\omega^2+K_1+K_2 & -(K_1 l_1 - K_2 l_2) \\ -(K_1 l_1 - K_2 l_2) & -J_G\omega^2 + K_1 l_1^2 + K_2 l_2^2 \end{bmatrix} \begin{bmatrix} X \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} (-1000\omega^2+40000) & 15000 \\ 150000 & (-810\omega^2+67500) \end{bmatrix} \begin{bmatrix} X \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \omega_1 = 5,86 \text{ rad.s}^{-1}$, $\omega_2 = 9,43 \text{ rad s}^{-1}$

La solution est la somme de deux solutions

$x(t) = X_{11} \cdot \cos(5,86 t + \varphi_1) + X_{12} \cdot \cos(9,43 \cdot t + \varphi_2)$

$\theta(t) = \theta_{11} \cos(5,86 \cdot t + \varphi_1) + \theta_{12} \cos(9,43 \cdot t + \varphi_2)$

Si le système oscille avec $\omega_1 = 5,86 \text{ rad.s}^{-1}$

$\Rightarrow \dfrac{X_{11}}{\theta_{11}} = -2,64$

Si le système oscille avec $\omega_2 = 9,43 \Rightarrow \dfrac{X_{12}}{\theta_{12}} = 0,31$

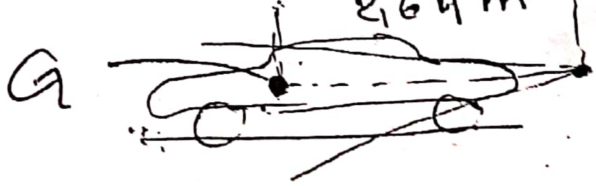
Donc :

$x(t) = X_{11} \cdot \cos(\omega_1 t + \varphi_1) + X_{12} \cos(\omega_2 t + \varphi_2)$

$\theta(t) = -2,64 X_{11} \cos(\omega_1 t + \varphi_1) + 0,31 X_{12} \cos(\omega_2 t + \varphi_2)$

X_{11}, X_{12}, φ_1 et φ_2 peuvent être trouvés des conditions initiales

$\omega = \omega_2$



Ex3

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