**Exercice 1.** Résoudre avec l’algorithme du simplexe en deux phases le programme linéaire (P).

$$P≡\left\{\begin{array}{c}Max(Z)=2x1+3x2+x3\\x1+x2+x3 \leq 40\\2x1+x2 - x3 \geq 10\\-x2+x3 \geq 10\\x1, x2, x3 \geq 0\end{array}\right.$$

1. forme standard.

$$P≡\left\{\begin{array}{c}Max(Z)=2x1+3x2+x3\\x1+x2+x3 +x4= 40\\2x1+x2 - x3 –x5= 10\\-x2+x3 –x6= 10\\x1, x2, x3 , x4, x5, x6\geq 0\end{array}\right.$$

1. forme artificielle.

$$P≡\left\{\begin{array}{c}Max(Z)=2x1+3x2+x3\\x1+x2+x3 +x4= 40\\2x1+x2 - x3 –x5+x7= 10\\-x2+x3 –x6+x8= 10\\x1, x2, x3 , x4, x5, x6\geq 0\end{array}\right.$$

2. **Phase I :**

**2.1 La table initiale**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | $$x1\downright $$ | **x2** | **x3** | **x4** | **x5** | **x6** | **x7** | **x8** | **bi** |
| **x4** | **1** | **1** | **1** | **1** | **0** | **0** | **0** | **0** | **40** |
| $\leftarrow $**x7** | **2** | **1** | **-1** | **0** | **-1** | **0** | **1** | **0** | **10** |
| **x8** | **0** | **-1** | **1** | **0** | **0** | **-1** | **0** | **1** | **10** |
| **W** | **2** | **0** | **0** | **0** | **-1** | **-1** | **0** | **0** | **20** |
| **Z** | **2** | **3** | **1** | **0** | **0** | **0** | **0** | **0** | **0** |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | $$x1$$ | **x2** | $$x3\downright $$ | **x4** | **x5** | **x6** | **x7** | **x8** | **bi** |
| **x4** | **0** | **1/2** | **3/2** | **1** | **1/2** | **0** | **-1/2** | **0** | **35** |
| **x1** | **1** | **1/2** | **-1/2** | **0** | **-1/2** | **0** | **1/2** | **0** | **5** |
| $\leftarrow $**x8** | **0** | **-1** | **1** | **0** | **0** | **-1** | **0** | **1** | **10** |
| **W** | **0** | **-1** | **1** | **0** | **0** | **-1** | **-1** | **0** | **10** |
| **Z** | **0** | **2** | **2** | **0** | **1** | **0** | **-1** | **0** | **-10** |

**1.2 Première itération**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | $$x1$$ | **x2** | $$x3$$ | **x4** | **x5** | **x6** | **x7** | **x8** | **bi** |
| **x4** | **0** | **2** | **0** | **1** | **1/2** | **3/2** | **-1/2** | **-3/2** | **20** |
| **x1** | **1** | **0** | **0** | **0** | **-1/2** | **-1/2** | **1/2** | **1/2** | **10** |
| **x3** | **0** | **-1** | **1** | **0** | **0** | **-1** | **0** | **1** | **10** |
| **W** | **0** | **0** | **0** | **0** | **0** | **0** | **-1** | **-1** | **0** |
| **Z** | **0** | **4** | **0** | **0** | **1** | **2** | **-1** | **-2** | **-30** |

**1.3 Deuxième itération**

La fonction objective atteint la valeur nulle et toutes les variables artificielles sont exclues de la base, donc on passe à la deuxième phase considérant la solution initiale retournée par la première.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | $$x1$$ | $$x2\downright $$ | $$x3$$ | **x4** | **x5** | **x6** | **bi** |
| $\leftarrow $**x4** | **0** | **2** | **0** | **1** | **1/2** | **3/2** | **20** |
| **x1** | **1** | **0** | **0** | **0** | **-1/2** | **-1/2** | **10** |
| **x3** | **0** | **-1** | **1** | **0** | **0** | **-1** | **10** |
| **Z** | **0** | **4** | **0** | **0** | **1** | **2** | **-30** |

**II. Phase II.**

**II.1 Table initiale**

**II.2 Première itération**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | $$x1$$ | $$x2$$ | $$x3$$ | **x4** | **x5** | **x6** | **bi** |
| **x2** | **0** | **1** | **0** | **1/2** | **1/4** | **3/4** | **10** |
| **x1** | **1** | **0** | **0** | **0** | **-1/2** | **-1/2** | **10** |
| **x3** | **0** | **0** | **1** | **1/2** | **1/4** | **-1/4** | **20** |
| **Z** | **0** | **0** | **0** | **-2** | **0** | **-1** | **-70** |

Tous les coefficients dans la ligne de la fonction objective sont négatifs ou nuls donc la solution est optimale avec :

x1\*=10, x2\*=10, x3\*=20, x4\*=x5\*=x6\*=0, Z\*=-Z=70.

**Exercice 2.** Résoudre avec la méthode du simplexe en deux phases :

 Maxz =x1-x2+x3

$$\left\{\begin{array}{c}2x1-x2-2x3\leq 4\\2x1-3x2+x3\leq -5\\-x1+x2-2x3\leq -1\\x1,x2, x3\geq 0\end{array}\right.$$

1) forme standard.

Maxz =x1-x2+x3

$$\left\{\begin{array}{c}2x1-x2-2x3+x4=4\\-2x1+3x2-x3-x5=5\\x1-x2+2x3-x6=1\\x1,x2, x3,x4,x5, x6\geq 0\end{array}\right.$$

**1.1 programme artificiel**

Maxz =x1-x2+x3

Max w =-w1-w2

$$\left\{\begin{array}{c}2x1-x2-2x3+x4=4\\-2x1+3x2-x3-x5+w1=5\\x1-x2+2x3-x6+w2=1\\x1,x2, x3,x4,x5, x6\geq 0\\w1\geq 0,w2\geq 0. \end{array}\right.$$

**2. Phase I :**

**2.1 La table initiale**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **X1** | **X2** | **X3** | **X4** | **X5** | **X6** | **W1** | **W2** | **bi** |
| **X4** | **2** | **-1** | **-2** | **1** | **0** | **0** | **0** | **0** | **4** |
| **W1** | **-2** | **3** | **-1** | **0** | **-1** | **0** | **1** | **0** | **5** |
| **W2** | **1** | **-1** | **2** | **0** | **0** | **-1** | **0** | **1** | **1** |
| **W** | **-1** | **2** | **1** | **0** | **-1** | **-1** | **0** | **0** | **6** |
| **Z** | **1** | **-1** | **1** | **0** | **0** | **0** | **0** | **0** | **0** |

La ligne de W est calculée en faisant la somme : LW=LW+Lw1+Lw2 afin d’exprimer W uniquement par les variables de base.

**2.2 Première itération :**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **X1** | **X2** | **X3** | **X4** | **X5** | **X6** | **W1** | **W2** | **bi** |
| **X4** | **4/3** | **0** | **-7/3** | **1** | **-1/3** | **0** | **1/3** | **0** | **17/3** |
| **X2** | **-2/3** | **1** | **-1/3** | **0** | **-1/3** | **0** | **1/3** | **0** | **5/3** |
| **W2** | **1/3** | **0** | **5/3** | **0** | **-1/3** | **-1** | **1/3** | **1** | **8/3** |
| **W** | **1/3** | **0** | **5/3** | **0** | **-1/3** | **-1** | **-2/3** | **0** | **8/3** |
| **Z** | **1/3** | **0** | **2/3** | **0** | **-1/3** | **0** | **1/3** | **0** | **5/3** |

**2.3 Deuxième itération**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **X1** | **X2** | **X3** | **X4** | **X5** | **X6** | **W1** | **W2** | **bi** |
| **X4** | **9/5** | **0** | **0** | **1** | **-4/5** | **-7/5** | **4/5** | **7/5** | **47/5** |
| **X2** | **-3/5** | **1** | **0** | **0** | **-2/5** | **-1/5** | **2/5** | **1/5** | **11/5** |
| **X3** | **1/5** | **0** | **1** | **0** | **-1/5** | **-3/5** | **1/5** | **3/5** | **8/5** |
| **W** | **0** | **0** | **0** | **0** | **0** | **0** | **-1** | **-1** | **0** |
| **Z** | **1/5** | **0** | **0** | **0** | **-1/5** | **2/5** | **1/5** | **2/5** | **3/5** |

**3. Phase II**

**3.1 Table initiale**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **X1** | **X2** | **X3** | **X4** | **X5** | **X6** | **bi** |
| **X4** | **9/5** | **0** | **0** | **1** | **-4/5** | **-7/5** | **47/5** |
| **X2** | **-3/5** | **1** | **0** | **0** | **-2/5** | **-1/5** | **11/5** |
| **X3** | **1/5** | **0** | **1** | **0** | **-1/5** | **-3/5** | **8/5** |
| **Z** | **1/5** | **0** | **0** | **0** | **-1/5** | **2/5** | **3/5** |

Tous les valeurs dans la colonne du pivot sont négatives ou nulles alors le problème admit une solution non-bornée.

**Exercice 3.** Résoudre avec la méthode du simplexe en deux phases :

Max z =3x1+x2

$$\left\{\begin{array}{c}x1-x2\leq 4\\-x1-x2\leq -3\\2x1+x2\leq 2\\x1,x2 \geq 0\end{array}\right.$$

**1. Problème artificiel**

Max z =3x1+x2

Max w=-w1

$$\left\{\begin{array}{c}x1-x2+x3=4\\x1+x2-x4+w1=3\\2x1+x2+x5=2\\x1,x2,x3,x4,x5 \geq 0\\w1\geq 0\end{array}\right.$$

**2. Phase I**

**2.1 Table initiale :**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **X1** | **X2** | **X3** | **X4** | **X5** | **W1** | **bi** |
| **X3** | **1** | **-1** | **1** | **0** | **0** | **0** | **4** |
| **W1** | **1** | **1** | **0** | **-1** | **0** | **1** | **3** |
| **X5** | **2** | **1** | **0** | **0** | **1** | **0** | **2** |
| **W** | **1** | **1** | **0** | **-1** | **0** | **0** | **3** |
| **Z** | **3** | **1** | **0** | **0** | **0** | **0** | **0** |

**2.2 Première itération**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **X1** | **X2** | **X3** | **X4** | **X5** | **W1** | **bi** |
| **X3** | **3** | **0** | **1** | **0** | **1** | **0** | **6** |
| **W1** | **-1** | **0** | **0** | **-1** | **-1** | **1** | **1** |
| **X2** | **2** | **1** | **0** | **0** | **1** | **0** | **2** |
| **W** | **-1** | **0** | **0** | **-1** | **-1** | **0** | **1** |
| **Z** | **1** | **0** | **0** | **0** | **-1** | **0** | **-2** |

Tous les coefficients dans la ligne LW sont négatifs alors que W1 est toujours dans la base et W n’attends pas encore sa valeur nulle donc le problème n’admet pas de solution du tout.

**Exercice 4.** Résoudre le programme linéaire par la méthode Big M.

Min z =3x1+4x2+5x3.

$$\left\{\begin{array}{c}x1+2x2+3x3\geq 5\\2x1+2x2+x3\geq 6\\\\x1,x2, x3\geq 0\end{array}\right.$$

**Le M-problème**

**Max Z=-3x1-4x2-5x3-mw1-mw2**

$$\left\{\begin{array}{c}x1+2x2+3x3-x4+w1=5\\2x1+2x2+x3-x5+w2=6\\\\x1,x2, x3, x4, x5,w1,w2\geq 0\end{array}\right.$$

1) Table initiale

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | x4 | x5 | w1 | w2 | bi |
| w1 | 1 | 2 | 3 | -1 | 0 | 1 | 0 | 5 |
| w2 | 2 | 2 | 1 | 0 | -1 | 0 | 1 | 6 |
| Z | -3+3m | -4+4m | -5+4m | -m | -m | 0 | 0 | 11m |

**1ière itération**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | x4 | x5 | w2 | bi |
| x1 | 1/2 | 1 | 3/2 | -1/2 | 0 | 0 | 5/2 |
| w2 | 1 | 0 | -2 | 1 | -1 | 1 | 1 |
| Z | m-1 | 0 | 1-2m | m-2 | -m | 0 | M+10 |

**2ième itération**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | x4 | x5 | bi |
| x2 | 0 | 1 | 5/2 | -1 | +1/2 | 2 |
| x1 | 1 | 0 | -2 | 1 | -1 | 1 |
| Z’ | 0 | 0 | -1 | -3 | -1 | 11 |

Tous les coefficients dans la ligne de la fonction objective sont négatifs ou nuls alors la solution est optimale avec :

x2\*=2, x1\*=1, Z\*=-Z=-11