**Exercice 1**

1) Le programme linéaire qui maximise la fonction Z est le suivant:

Max Z=x1-x2+x3

$$\left\{\begin{array}{c}x1+x2+x3\leq 8\\x2\leq 1\\\\x1,x2, x3\geq 0\end{array}\right.$$

1.1- La résolution du problème à l’aide de la méthode du simplexe :

 a) la forme standard :

Max Z=x1-x2+x3

$$\left\{\begin{array}{c}x1+x2+x3+x4=8\\x2+x5=1\\\\x1,x2, x3, x4,x5 \geq 0\end{array}\right.$$

b) La table initiale

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| V.B | x1 $\downright $ | x2 | x3 | x4 | x5 | bi |
| $\leftarrow $x4 | 1 | 1 | 1 | 1 | 0 | 8 |
| x5 | 0 | 1 | 0 | 0 | 1 | 1 |
| Z | 1 | -1 | 1 | 0 | 0 | 0 |

b) Première itération

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| V.B | x1 | x2 | x3 | x4 | x5 | bi |
| x1 | 1 | 1 | 1 | 1 | 0 | 8 |
| x5 | 0 | 1 | 0 | 0 | 1 | 1 |
| Z | 0 | -2 | 0 | -1 | 0 | -8 |

Tous les coefficients dans la ligne de la fonction objective sont négatifs ou nuls alors la fonction Z est maximale avec :

X1\*=8, X2\*=X3\*=X4\*=0, X5\*=1 avec Z\*=8.

2) Le programme linéaire qui minimise la fonction Z est le suivant:

Min Z=x1-x2+x3

$$\left\{\begin{array}{c}x1+x2+x3\leq 8\\x2\leq 1\\\\x1,x2, x3\geq 0\end{array}\right.$$

2.1 La résolution du problème à l’aide de la méthode du simplexe :

 a) la forme standard :

Max W=Min -Z =-x1+x2-x3

$$\left\{\begin{array}{c}x1+x2+x3+x4=8\\x2+x5=1\\\\x1,x2, x3, x4,x5 \geq 0\end{array}\right.$$

b) La table initiale

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| V.B | x1  | x2 $\downright $ | x3 | x4 | x5 | bi |
| x4 | 1 | 1 | 1 | 1 | 0 | 8 |
| $\leftarrow $x5 | 0 | 1 | 0 | 0 | 1 | 1 |
| W | -1 | 1 | -1 | 0 | 0 | 0 |

b) Première itération

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| V.B | x1 | x2 | x3 | x4 | x5 | bi |
| x4 | 1 | 0 | 1 | 1 | -1 | 7 |
| X2 | 0 | 1 | 0 | 0 | 1 | 1 |
| W | -1 | 0 | -1 | 0 | -1 | -1 |

Tous les coefficients dans la ligne de la fonction objective sont négatifs ou nuls alors la fonction W est maximale avec :

X2\*=1, X4\*=7, X1\*=X3\*=X5\*=0, avec MAXW=1$⟹$Min Z=1

**Exercice 2**

Max Z=4x1+12x2+3x3

$$\left\{\begin{array}{c}x1\leq 1000\\x2\leq 500\\x3\leq 1500\\3x1+6x2+2x3\leq 1500\\x1,x2, x3\geq 0\end{array}\right.$$

Forme standard

Max Z=4x1+12x2+3x3

$$\left\{\begin{array}{c}x1+x4=1000\\x2+x5=500\\x3+x6=1500\\3x1+6x2+2x3+x7=1500\\x1,x2, x3\geq 0\\x4,x5, x6\geq 0\end{array}\right.$$

Table initiale

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **x1** | **x2** | **x3** | **x4** | **x5** | **x6** | **x7** | **bi** |
| **x4** | **1** | **0** | **0** | **1** | **0** | **0** | **0** | **1000** |
| $\leftarrow $**x5** | **0** | **1** | **0** | **0** | **1** | **0** | **0** | **500** |
| **x6** | **0** | **0** | **1** | **0** | **0** | **1** | **0** | **1500** |
| **X7** | **3** | **6** | **2** | **0** | **0** | **0** | **1** | **6750** |
| **Z** | **4** | **3** | **12** | **0** | **0** | **0** | **0** | **0** |

**1ere itération**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | $$x\_{1}\downright $$ | **x2** | **x3** | **x4** | **x5** | **x6** | **x7** | **bi** |
| $\leftarrow $**x4** | **1** | **0** | **0** | **1** | **0** | **0** | **0** | **1000** |
| **x5** | **0** | **1** | **0** | **0** | **1** | **0** | **0** | **500** |
| **x6** | **0** | **0** | **1** | **0** | **0** | **1** | **0** | **1500** |
| **X7** | **3** | **0** | **2** | **0** | **-6** | **0** | **1** | **6750** |
| **Z** | **4** | **0** | **3** | **0** | **-12** | **0** | **0** | **-6000** |

**2eme itération**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | $$x\_{1}$$ | **x2** | $$x\_{3}\downright $$ | **x4** | **x5** | **x6** | **x7** | **bi** |
| **x1** | **1** | **0** | **0** | **1** | **0** | **0** | **0** | **1000** |
| **x2** | **0** | **1** | **0** | **0** | **1** | **0** | **0** | **500** |
| **x6** | **0** | **0** | **1** | **0** | **0** | **1** | **0** | **1500** |
| $\leftarrow $**x7** | **0** | **0** | **2** | **-3** | **-6** | **0** | **1** | **750** |
| **Z** | **4** | **0** | **3** | **0** | **-12** | **0** | **0** | **-10000** |

**3eme itération**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | $$x\_{1}$$ | **x2** | $$x\_{3}$$ | $$x\_{4}\downright $$ | **x5** | **x6** | **x7** | **bi** |
| **X1** | **1** | **0** | **0** | **1** | **0** | **0** | **0** | **1000** |
| **x2** | **0** | **1** | **0** | **0** | **1** | **0** | **0** | **500** |
| $\leftarrow $**x6** | **0** | **0** | **0** | **3/2** | **3** | **1** | **-1/2** | **1500** |
| **x3** | **0** | **0** | **1** | **-3/2** | **-3** | **0** | **1/2** | **375** |
| **Z** | **0** | **0** | **0** | **1/2** | **-3** | **0** | **-3/2** | **-11125** |

**4eme itération**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | $$x\_{1}$$ | **x2** | $$x\_{3}$$ | $$x\_{4}$$ | **x5** | **x6** | **x7** | **bi** |
| **x1** | **1** | **0** | **0** | **0** | **-2** | **-2/3** | **-1/3** | **250** |
| **x2** | **0** | **1** | **0** | **0** | **1** | **0** | **0** | **500** |
| **x4** | **0** | **0** | **0** | **1** | **2** | **2/3** | **-1/3** | **750** |
| **x3** | **0** | **0** | **1** | **0** | **0** | **1** | **0** | **1500** |
| **Z** | **0** | **0** | **0** | **0** | **-4** | **-1/3** | **-4/3** | **-11500** |

Tous les coefficients dans la ligne de la fonction objective sont négatifs ou nuls alors, la solution est optimale avec :

x1\*=250, x2\*=500, x3\*=1500, x4\*=750, x5\*=x6\*=x7\*=0, Z\*=-Z=11500.

 **Exercice 3** Résoudre avec la méthode du simplexe :

 Max Z= 3x1+2x2+4x3

$$\left\{\begin{array}{c}x1+x2+2x3 \leq 4\\2x1+3x3\leq 5\\2x1+x2+3x3\leq 7\\x\&1 , x2,x3\geq 0\end{array}\right.$$

**1- La forme standard du PL :**

Max Z= 3x1+2x2+4x3

$$\left\{\begin{array}{c}x1+x2+2x3+x3=4\\2x1+3x3+x5=5\\2x1+x2+3x3+x6=7\\x\&1 , x2,x3, x4,x5,x6\geq 0\end{array}\right.$$

**2- Table initiale**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **bi** | **x1** | **x2** | **x3**$\downright $ | **x4** | **x5** | **x6** |
| **x4** | **4** | **1** | **1** | **2** | **1** | **0** | **0** |
| $\leftarrow $**x5** | **5** | **2** | **0** | **3** | **0** | **1** | **0** |
| **x6** | **7** | **2** | **1** | **3** | **0** | **0** | **1** |
| **Z** | **0** | **3** | **2** | **4** | **0** | **0** | **0** |

**3- Première itération**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **bi** | **x1** | **x2**$\downright $ | **x3** | **x4** | **x5** | **x6** |
| $\leftarrow $**x4** | **2/3** | **-1/3** | **1** | **0** | **1** | **-2/3** | **0** |
| **x3** | **5/3** | **2/3** | **0** | **1** | **0** | **1/3** | **0** |
| **x6** | **2** | **0** | **1** | **0** | **0** | **-1** | **1** |
| **Z** | **-20/3** | **1/3** | **2** | **0** | **0** | **-4/3** | **0** |

**4- Deuxième itération**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **bi** | **x1**$\downright $ | **x2** | **x3** | **x4** | **x5** | **x6** |
| **x2** | **2/3** | **-1/3** | **1** | **0** | **1** | **-2/3** | **0** |
| $\leftarrow $**x3** | **5/3** | **2/3** | **0** | **1** | **0** | **1/3** | **0** |
| **x6** | **4/3** | **1/3** | **0** | **0** | **-1** | **-1/3** | **1** |
| **Z** | **-8** | **1** | **0** | **0** | **-2** | **0** | **0** |

**5- Troisième itération**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **bi** | **x1** | **x2** | **x3** | **x4** | **x5** | **x6** |
| **x2** | **3/2** | **0** | **1** | **1/2** | **1** | **-1/2** | **0** |
| **x1** | **5/2** | **1** | **0** | **3/2** | **0** | **1/2** | **0** |
| **x6** | **1/2** | **0** | **0** | **-1/2** | **-1** | **-1/2** | **1** |
| **Z** | **-21/2** | **0** | **0** | **-3/2** | **-2** | **-1/2** | **0** |

On constate que tous les coefficients dans la ligne de la fonction objective sont négatifs ou nuls alors, la solution est optimale avec :

x1\*=x1=5/2, x2\*=x2=3/2, x6\*=x6=3/2, x3\*=x4\*=x5\*=0.

Z\*=-Z=21/2.

**Exercice 4.**

Max z =-5x1+5x2+13x3

$$\left\{\begin{array}{c}-x1+x2+3x3\leq 20\\12x1+4x2+10x3\leq 90\\x1,x2, x3\geq 0\end{array}\right.$$

Max z =-5x1+x2+13x3

$$\left\{\begin{array}{c}-x1+x2+3x3+x4=20\\12x1+4x2+10x3+x5=90\\x1,x2, x3,x4, x5\geq 0\end{array}\right.$$

1) Tableau initial :

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **x1** | **x2** | **x3**$\downright $ | **x4** | **x5** | **Bi** |
| $\leftarrow $**x4** | **-1** | **1** | **3** | **1** | **0** | **20** |
| **x5** | **12** | **4** | **10** | **0** | **1** | **90** |
| **Z** | **-5** | **5** | **13** | **0** | **0** | **0** |

2) Première itération

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **x1** | **X2**$\downright $ | **x3** | **x4** | **x5** | **Bi** |
| $\leftarrow $**x3** | **-1/3** | **1/3** | **1** | **1/3** | **0** | **20/3** |
| **x5** | **46/3** | **2/3** | **0** | **-10/3** | **1** | **70/3** |
| **Z** | **-2/3** | **2/3** | **0** | **-13/3** | **0** | **-260/3** |

2) Deuxième itération

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **x1** | **x2** | **x3**$\downright $ | **x4** | **x5** | **Bi** |
| $$x3$$ | **-1** | **1** | **3** | **1** | **0** | **20** |
| **x5** | **16** | **0** | **-2** | **-4** | **1** | **10** |
| **Z** | **0** | **0** | **-2** | **-5** | **0** | **-100** |

Tous les coefficients dans la ligne de la fonction objective sont négatifs ou nuls alors, la solution est optimale avec :

x2\*=x2=20, x5\*=x5=10, x1\*=x3\*=x4\*=0.

Z\*=-Z=100

2- La nouvelle base en changeant le second membre de la contrainte n 1 :

B=$\left(\begin{array}{c}x2\\x5\end{array}\right)$ , AB=$\left(\begin{matrix}1&0\\4&1\end{matrix}\right)$ AB-1=$\left(\begin{matrix}1&0\\-4&1\end{matrix}\right)$

$\left(\begin{array}{c}x2\\x5\end{array}\right)$=$\left(\begin{matrix}1&0\\-4&1\end{matrix}\right)$ $\left(\begin{array}{c}30\\90\end{array}\right)$= $\left(\begin{array}{c}30\\-30\end{array}\right)$ x5<0 Solution non admissible.

2.1 La solution du problème par l algorithme du simplexe.

. 1) Tableau initial :

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **x1** | **x2** | **x3**$\downright $ | **x4** | **x5** | **Bi** |
| **x4** | **-1** | **1** | **3** | **1** | **0** | **30** |
| $\leftarrow $**x5** | **12** | **4** | **10** | **0** | **1** | **90** |
| **Z** | **-5** | **5** | **13** | **0** | **0** | **0** |

2) Première itération

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **v.b** | **x1** | **X2**$\downright $ | **x3** | **x4** | **x5** | **Bi** |
| **x4** | **-4,6** | **-0,2** | **0** | **1** | **-0,3** | **3** |
| **x3** | **1,2** | **0,4** | **1** | **0** | **0,1** | **9** |
| **Z** | **-20,6** | **-0.2** | **0** | **0** | **-1.3** | **-117** |

Tous les coefficients dans la ligne de la fonction objective sont négatifs ou nuls alors, la solution est optimale avec :

x3\*=x3=9, x4\*=x4=3, x1\*=x2\*=x5\*=0.

Z\*=-Z=117.

L intervalle de validité de la base B=$\left(\begin{array}{c}x2\\x5\end{array}\right) $ :

$\left(\begin{matrix}1&0\\-4&1\end{matrix}\right)$ $\left(\begin{array}{c}b\\90\end{array}\right)$=$\left(\begin{array}{c}b\\-4b+90\end{array}\right)\geq \left(\begin{array}{c}0\\0\end{array}\right)$ => b$\in [0, 22.5]$.