Ex 01 :

① $\frac{x^{n+1}}{n+1}$ ② $\frac{x^{\alpha+1}}{\alpha+1}$ ③ $-\cos x$ ④ $\sin x$

⑤ $\tan x$ ⑥ e^x ⑦ $\ln|x|$ ⑧ $\arcsin x$

⑨ $\arccos x$ ⑩ $\arctan x$.

$\tan x = \frac{\sin x}{\cos x}$

$\frac{\cos x^2 + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} =$

$1 + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2$

Ex 2 :

$I_1 = \begin{cases} u(x) = \arctan x \\ v'(x) = 1 \end{cases}$ $u' = \frac{1}{x^2+1}$, $v(x) = x$ $\Rightarrow$ $x \arctan x - \int \frac{x}{x^2+1} dx$

$\ln|x^2+1|$

$I_1 = x \arctan x - \frac{1}{2} \int \frac{2x}{x^2+1} = x \arctan x - \frac{1}{2} \ln(x^2+1) + C$

$I_2 = \begin{cases} u(x) = (\ln x)^2 \\ v'(x) = 1 \end{cases}$ $u' = 2\frac{\ln x}{x}$, $v = x$ $\Rightarrow$ $x(\ln x)^2 - 2\int \ln x \, dx$

$= x(\ln x)^2 - 2x\ln x + 2x + C$ $\quad [\int \ln x \quad \ln x \quad 1 \quad \frac{1}{x} \quad x = x\ln x - \int 1 = x\ln x - x$

$I_3 = \begin{cases} u = x \\ v' = \sin x \end{cases}$ $u' = 1$, $v = -\cos x$ $\Rightarrow$ $-x\cos x + \int \cos x$

$= -x\cos x + \sin x + C$

$I_4 = \begin{cases} u = x^2 \\ v' = \cos x \end{cases}$ $u' = 2x$, $v = \sin x$ $\Rightarrow$ $x^2 \sin x - \int 2x \sin x \, dx$

$x^2 \sin x - 2\int I_3 \, dx$

$= x^2 \sin x - 2[-x\cos x + \sin x] + C$

$I_5 = \int x e^x dx$ $\begin{cases} u' = e^x \\ v = x \end{cases}$, $u = e^x$, $v' = 1$ $\Rightarrow$ $xe^x - \int e^x = xe^x - e^x = e^x(x-1) + C$

$I_6 = \int x \ln x \, dx \Rightarrow \begin{cases} u = \ln x \\ v' = x \end{cases}$ $u' = \frac{1}{x}$, $v = \frac{x^2}{2}$ $\Rightarrow$ $\frac{x^2}{2}\ln x - \frac{1}{2}\int x \, dx$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

Ex3 :

$I_1 = \int_1^4 \frac{1-\sqrt{t}}{\sqrt{t}} dt \Rightarrow u = \sqrt{t} \Rightarrow t = u^2 \Rightarrow dt = 2u\,du.$

$t = 1 \Rightarrow u = 1 \mid t = 4 \Rightarrow u = 2$

$I_1 = \int_1^2 \frac{1-u}{\cancel{u}} \cdot 2\cancel{u}\, du = 2\int_1^2 1-u\,du = 2\left[u - \frac{u^2}{2}\right]_1^2$

$= 2\left[2 - 2 - 1 + \frac{1}{2}\right] = -1.$

$I_2 = \int_1^2 \frac{e^x}{1+e^x} dx \Rightarrow u = e^x \Rightarrow x = \ln u \Rightarrow dx = \frac{1}{u} du.$

$x = 1 \Rightarrow u = e \mid x = 2 \Rightarrow u = e^2$

$I_2 = \int_e^{e^2} \frac{\cancel{u}}{1+u} \cdot \frac{1}{\cancel{u}} du = \int_e^{e^2} \frac{1}{1+u} du = \ln|1+u|\Big|_e^{e^2}$

$= \ln|1+e^2| - \ln|1+e| = \ln\left(\frac{1+e^2}{1+e}\right).$

$I_3 = \int_1^e \frac{(\ln x)^n}{x} dx \Rightarrow u = \ln x \Rightarrow x = e^u \quad dx = e^u du.$

$x = 1 \Rightarrow u = 0 \mid x = e \Rightarrow u = 1.$

$I_3 = \int_0^1 \frac{u^n}{\cancel{e^u}} \cdot \cancel{e^u} du = \int_0^1 u^n = \frac{u^{n+1}}{n+1}\Big|_0^1 = \frac{1}{n+1}.$

$I_4 = \int \frac{\ln x}{x} dx \Rightarrow u = \ln x \Rightarrow x = e^u. \quad dx = e^u du.$

$I_4 = \int \frac{u}{\cancel{e^u}} \cdot \cancel{e^u} du. = \int u\,du = \frac{u^2}{2} + C$

$I_4 = \frac{1}{2}(\ln x)^2 + C.$

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Ex 04 :

$$I_1 = \int_0^{\frac{\pi}{4}} \frac{\tan x}{1+\cos x}\, dx = \int \frac{\sin x}{\cos x(1+\cos x)}\, dx.$$

- $u = \cos x \Rightarrow du = -\sin x\, dx. \Rightarrow dx = -\frac{1}{\sin x}\, du.$

$$I_1 = -\int \frac{\cancel{\sin x}}{u(1+u)} \cdot \frac{1}{\cancel{\sin x}}\, du = -\int \frac{1}{u(1+u)}\, du.$$

$$\frac{1}{u(1+u)} = \frac{A}{u} + \frac{B}{1+u} \Rightarrow A + Au + Bu = A + u(A+B) = 1$$

$$\Rightarrow A + B = 0 \Rightarrow A = -B \quad \text{And} \quad A = 1 \Rightarrow B = -1$$

$$I_1 = -\left[\int \frac{1}{u}\, du - \int \frac{1}{1+u}\, du\right] = -\ln|u| + \ln|1+u|$$

$$= \ln\left|\frac{1+u}{u}\right| + C \Rightarrow I_1 = \ln\left|\frac{1+\cos x}{\cos x}\right| + C.$$

$$I_2 = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x - 5\sin x + 6}\, dx. \quad \Big| \quad u = \sin x$$

$$du = \cos x\, dx \Rightarrow dx = \frac{1}{\cos x}\, du$$

$x = 0 \Rightarrow u = 0$

$x = \frac{\pi}{2} \Rightarrow u = 1.$

$$\int_0^1 \frac{\cancel{\cos x}}{u^2 - 5u + 6} \cdot \frac{1}{\cancel{\cos x}}\, du = \int_0^1 \frac{1}{u^2 - 5u + 6}\, du. \qquad u_{1,2} = 3, 2.$$

$$= \int_0^1 \frac{1}{(u-3)(u-2)}\, du. = \frac{A}{u-3} + \frac{B}{u-2}, \quad A = 1, \quad B = -1$$

$$= \int_0^1 \frac{1}{u-3} - \frac{1}{u-2}\, du = \ln|u-3| - \ln|u-2| \Big]_0^1$$

$$= \ln\left|\frac{u-3}{u-2}\right|\Bigg|_0^1 = \ln(2) - \ln\frac{3}{2}.$$

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$$I_3 = \int \frac{\sqrt[4]{1+x^3}}{x}\,dx$$

$$u = \sqrt[4]{1+x^3} \Rightarrow u^4 = 1+x^3 \Rightarrow x = \sqrt[3]{u^4+1} = (u^4+1)^{\frac{1}{3}} \Rightarrow dx = \frac{4}{3}(u^4-1)^{-\frac{2}{3}}u^3\,du$$

$$\int \frac{\sqrt[4]{1+x^3}}{x}\,dx = \int \frac{u}{\sqrt[3]{u^4-1}} \cdot \frac{4}{3}\,u^3\,(u^4-1)^{-\frac{2}{3}}\,du = \frac{4}{3}\int \frac{u^4}{u^4-1}\,du.$$

$$= \frac{4}{3}\int 1 + \frac{1}{u^4-1}\,du$$

$$u^4-1 = (u^2)^2-1 = \overbrace{(u+1)(u-1)}^{(u^2-1)}(u^2+1)$$

$$\frac{1}{u^4-1} = \frac{a}{u-1} + \frac{b}{u+1} + \frac{cu+d}{u^2+1}$$

$$\begin{cases} a \to u^3+u^2+u+1 \\ b \to u^3-u^2+u-1 \end{cases}$$
$$cu^3+du^2-cu-d$$

$$\begin{aligned} &(1)\ a+b+c=0 \\ &(2)\ a-b+d=0 \\ &(3)\ a+b-c=0 \\ &(4)\ a-b-d=1 \end{aligned} \quad \begin{cases} (1)+(3) \Rightarrow a=-b \\ (2)+(4) \Rightarrow b=-\frac{1}{4} \\ c=0,\ d=-\frac{1}{2} \end{cases}$$

$$= \int 1 + \frac{1}{4(u-1)} - \frac{1}{4(u+1)} - \frac{1}{2}\,\frac{1}{u^2+1}\,du.$$

$$= u + \frac{1}{4}\ln|u-1| - \frac{1}{4}\ln|u+1| - \frac{1}{2}\arctan u.\ \cancel{+C}$$

$$= u + \frac{1}{4}\ln\left|\frac{u-1}{u+1}\right| - \frac{1}{2}\arctan u.\ \cancel{+C}$$

$$I_3 = \frac{4}{3}\int 1 + \frac{1}{u^4-1}\,du = \frac{4}{3}u + \frac{1}{3}\ln\left|\frac{u-1}{u+1}\right| - \frac{2}{3}\arctan u + C$$

$$= \frac{4}{3}\sqrt[4]{1+x^3} + \frac{1}{3}\ln\left|\frac{\sqrt[4]{1+x^3}-1}{\sqrt[4]{1+x^3}+1}\right| - \frac{2}{3}\arctan\left(\sqrt[4]{1+x^3}\right) + C$$