حل سلسلة تمارين رقم 04

التمرين الأول

حساب التكاملات

1) $n \geq 1$, $\int \underbrace{x^n}_{u'} \underbrace{Lnx}_{v}\, dx = \frac{1}{n+1}x^{n+1} - \frac{1}{n+1}\int x^{n+1}.\frac{1}{x}dx$

$= \frac{1}{n+1}x^{n+1} - \frac{1}{(n+1)^2}x^{n+1} + c = \frac{1}{n+1}x^{n+1}\left[1 - \frac{1}{n+1}\right] + c$

2) $\int \underbrace{x^2}_{u} \underbrace{e^x}_{v'} dx = x^2e^x - 2\int \underbrace{x}_{f} \underbrace{e^x}_{g'} dx = x^2e^x - 2[xe^x - \int e^x dx]$

$= x^2e^x - 2[xe^x - e^x] + c = e^x[x^2 - 2x + 2] + c$

3) $\int \frac{x^3}{\sqrt[3]{x^2+4}}dx$

نضع $\sqrt[3]{x^2+4} = y$ إذن $x^2 = y^3 - 4$, $\left[(x^2+4)^{\frac{1}{3}}\right]^{\backslash} dx = 1.dy$

$$\Rightarrow \frac{1}{3}2x(x^2+4)^{-\frac{2}{3}}dx = dy \Rightarrow \frac{2}{3}x\frac{1}{y^2}dx = dy \Rightarrow dx = \frac{3y^2}{2x}dy$$

$$\int \frac{x^3}{\sqrt[3]{x^2+4}}dx = \frac{3}{2}\int \frac{x^3y^2}{xy}dy = \frac{3}{2}\int x^2y\,dy = \frac{3}{2}\int (y^3-4)y\,dy$$

$$= \frac{3}{2}\int (y^4 - 4y)\,dy = \frac{3}{2}\left(\frac{1}{5}y^5 - \frac{4}{2}y^2\right) + c$$

$$= \frac{3}{10}(x^2+4)^{\frac{5}{3}} - 3(x^2+4)^{\frac{2}{3}} + c$$

التمرين الثاني

حساب التكاملات

1) $\int \frac{1}{x^2-5x+6}dx$, $(\Delta > 0\ ,\ x_1 = 2\ ,\ x_2 = 3)$

$$\frac{1}{(x-2)(x-3)} = \frac{a}{x-2} + \frac{b}{x-3}$$

لإيجاد a نضرب$(x-2)$ ثم نعوض بـ $(x=2)$ نجد في

$$\frac{1}{2-3} = a \qquad \Rightarrow a = -1$$

لإيجاد b نضرب$(x-3)$ ثم نعوض بـ $(x=3)$ نجد في

$$\frac{1}{3-2} = b \qquad \Rightarrow b = 1$$

إذن

$$\int \frac{1}{x^2-5x+6}dx = \int \frac{1}{(x-2)(x-3)}dx = -\int \frac{1}{x-2}dx + \int \frac{1}{x-3}dx$$

$$= -Ln|x-2| + Ln|x-3| + c$$

2) $\int \frac{1}{\left(\sqrt{2}x^2-2x+\frac{\sqrt{2}}{2}\right)^2} dx$, $\left(\Delta = 0\ ,\ x_0 = \frac{1}{\sqrt{2}}\right)$

$$\int \frac{1}{\left(\sqrt{2}x^2-2x+\frac{\sqrt{2}}{2}\right)^2} dx = \frac{1}{\sqrt{2}}\int \frac{1}{\left(x-\frac{1}{\sqrt{2}}\right)^2} dx = \frac{1}{\sqrt{2}}\frac{-1}{\left(x-\frac{1}{\sqrt{2}}\right)} + c$$

$$= \frac{-1}{(\sqrt{2}x-1)} + c$$

3) $\int \frac{1}{2x^2-x+3} dx$, $(\Delta = -23)$

$$2x^2 - x + 3 = (\sqrt{2}x)^2 - 2(\sqrt{2}x)\frac{1}{2\sqrt{2}} + \left(\frac{1}{2\sqrt{2}}\right)^2 + \left[3 - \left(\frac{1}{2\sqrt{2}}\right)^2\right]$$

$$= \left(\sqrt{2}x - \frac{1}{2\sqrt{2}}\right)^2 + \left[\frac{23}{8}\right]$$

$$= \frac{23}{8}\left[\left[\frac{\sqrt{8}}{\sqrt{23}}\left(\sqrt{2}x - \frac{1}{2\sqrt{2}}\right)\right]^2 + 1\right]$$

إذن

1) $$\int \frac{1}{2x^2-x+3} dx = \frac{8}{23}\int \frac{1}{\left[\left[\frac{\sqrt{8}}{\sqrt{23}}\left(\sqrt{2}x-\frac{1}{2\sqrt{2}}\right)\right]^2+1\right]} dx = \frac{\sqrt{8}}{\sqrt{23}}\int \frac{\frac{\sqrt{8}}{\sqrt{23}}}{\left[\left[\frac{\sqrt{8}}{\sqrt{23}}\left(\sqrt{2}x-\frac{1}{2\sqrt{2}}\right)\right]^2+1\right]} dx$$

$$= \frac{\sqrt{8}}{\sqrt{23}} arctg\left[\frac{\sqrt{8}}{\sqrt{23}}\left(\sqrt{2}x - \frac{1}{2\sqrt{2}}\right)\right] + c$$

التمرين الثالث

إيجاد I و J حيث $I = \int_0^{\frac{\pi}{4}} x sin^2x\, dx$ و $J = \int_0^{\frac{\pi}{4}} x cos^2x\, dx$

لنحسب $I + J$ و $I - J$

$$I + J = \int_0^{\frac{\pi}{4}} x(sin^2x + cos^2x)\, dx = \int_0^{\frac{\pi}{4}} x\, dx = \left[\frac{1}{2}x^2\right]_0^{\frac{\pi}{4}} = \frac{\pi^2}{32}$$

$$I - j = \int_0^{\frac{\pi}{4}} x(sin^2x - cos^2x)\, dx = -\int_0^{\frac{\pi}{4}} x(cos^2x - sin^2x)\, dx$$

$$= -\int_0^{\frac{\pi}{4}} \underbrace{x}_{u}\, \underbrace{cos2x}_{v'}\, dx = -\left[\frac{1}{2}x sin2x\right]_0^{\frac{\pi}{4}} + \frac{1}{2}\int_0^{\frac{\pi}{4}} sin2x\, dx = -\frac{\pi}{8} - \frac{1}{4}\underbrace{[cos2x]_0^{\frac{\pi}{4}}}_{-1} = \frac{1}{4} - \frac{\pi}{8}$$

$$\begin{cases} I + J = \frac{\pi^2}{32} & \dots(1) \\ I - j = \frac{1}{4} - \frac{\pi}{8} & \dots(2) \end{cases} \qquad (1) + (2) \Longrightarrow\ 2I = \frac{\pi^2}{32} - \frac{\pi}{8} + \frac{1}{4} \Longrightarrow I = \frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2}$$

$$(2) \qquad \Longrightarrow \qquad j = I + \frac{\pi}{8} - \frac{1}{4} \Longrightarrow J = \frac{\pi^2}{16} - \frac{\pi}{8} + \frac{1}{4}$$