

Institut des Sciences et de la Technologie  
Matière: Equation de la physique mathématique.

## Série N°2

### Exercice 1

Montrer que l'EDP suivante est linéaire :

- 1)  $\frac{\partial^2 u}{\partial x^2} + x \frac{\partial^2 u}{\partial y^2} = 0$
- 2)  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + 4 \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0$

### Exercice 2

Montrer que la fonction  $u(x, y) = x^2 - y^2$  est une solution d'équation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

### Exercice 3 :

Déterminer la classe et les courbes caractéristiques d'équations suivantes :

- 1)  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = f.$
- 2)  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f.$
- 3)  $\frac{\partial^2 u}{\partial x^2} + x \frac{\partial^2 u}{\partial y^2} = f.$
- 4)  $x^2 \frac{\partial^2 u}{\partial x^2} - y^2 \frac{\partial^2 u}{\partial y^2} = 0.$
- 5)  $x^4 \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0.$
- 6)  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = f$

### Exercice 4 :

Déterminer la forme canonique d'équations suivantes :

- 1)  $\frac{\partial^2 u}{\partial x^2} + (1 + y)^2 \frac{\partial^2 u}{\partial y^2} = 0 .$
- 2)  $\frac{\partial^2 u}{\partial x^2} + x \frac{\partial^2 u}{\partial y^2} = 0.$
- 3)  $\frac{\partial^2 u}{\partial x^2} - 3 \frac{\partial^2 u}{\partial x \partial y} + 4 \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0.$
- 4)  $\frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$